

# **On the 'Lock-In' Effects of Capital Gains Taxation**

Yoshitsugu Kanemoto<sup>1</sup>

Faculty of Economics, University of Tokyo

7-3-1 Hongo, Bunkyo-ku, Tokyo 113 Japan

## **Abstract**

The most important drawback of a tax on realized capital gains is its "lock-in" effect. This paper uses a simple land development model to examine the distortion that the lock-in effect generates. A surprising result is that the lock-in effect does not arise if the basis for the capital gains tax (usually the price at which the current owner acquired the land) is sufficiently high. Rather than delaying the sale, the owner sells the land as soon as possible even if the land will be developed much later. In this case, the capital gains tax creates no "real" distortion because it does not affect the development time. In particular, if the basis is the price formed under perfect foresight, the lock-in effect never arises.

Proposed running head: Lock-In Effects of Capital Gains Taxation

## 1. Introduction

The most important drawback of taxing capital gains only upon realization rather than on accrual is its "lock-in" effect. Because taxes are deferred until the asset is sold, investors tend to be locked into previously purchased assets. How big a distortion that the lock-in effect creates cannot be evaluated, however, unless we have a model that links asset holdings to real resource allocation. This paper uses a simple land development model to examine the distortion.

Consider a plot of land that is currently under-utilized, for example, a farm located at a commuting distance from the city center. Converting it into residential use yields much higher rents, but it requires substantial development and construction costs. The lock-in effect will distort the "real" resource allocation if it delays development.

The lock-in effect does not cause any distortion if development does not require the sale of land. If a rental contract were as efficient as the sale of the land, for example, the owner could rent the land to a household that would build its own house on the rented land. The rental arrangement is often inefficient, however, because the house cannot be separated from the land. As well known in the human capital literature pioneered by Becker [2], the specificity of capital generates serious market failure when a complete contingent contract is not feasible. It is often the case, therefore, that the outright sale of land is far superior to the rental contract. If the sale of land is essential for development, the lock-in effect results in a distortion in real resource allocation because it delays development.

A surprising result in this model is that the lock-in effect does not arise if the basis for capital gains taxation (usually the price at which the current owner acquired the land)

is sufficiently high. In fact, the opposite extreme is obtained. Rather than delaying the sale, the owner sells the land immediately even if development occurs much later. The capital gains tax creates no "real" distortion in this case because it does not distort the buyer's choice of development timing. If the basis is low, however, the owner sells the land just before development and the development time is in general distorted. A sufficient condition for the non-existence of the lock-in effect is that the basis is the purchase price that was formed under perfect foresight. The lock-in effect arises only when unexpected capital gains exist.

The organization of this paper is as follows. A model of land development is formulated in Section 2. Section 3 solves for the optimal timing of sale given an arbitrary development time. The optimal solution is either at the initial time or just before development. Section 4 first derives conditions for optimal development timing. Combining these conditions with results in Section 3 shows that the owner chooses to sell the land at the initial time when the purchase price is sufficiently high. A corollary of this result is that the lock-in effect arises only when unexpected capital gains exist. Section 5 contains concluding discussions.

## **2. The Model**

Consider a plot of land that is currently undeveloped (or under-utilized). If it remains undeveloped, the land rent is  $v(t)$  at time  $t$ , where  $v(t) \geq 0$ . The costs of developing the land and constructing a building are constant over time and denoted by  $C$ . The building is infinitely durable and cannot be demolished. After development the property yields gross rents  $R(t)$  at time  $t$ . The net land rent is the gross rents minus the user cost of capital. Under our assumption of perfect durability of capital, the user cost

is  $iC$  when the (after-tax) interest rate is  $i$ . The net rent is then  $r(t) = R(t) - iC$ .

We assume perfect foresight of the rent profiles. Furthermore, the rent profiles satisfy the following conditions. First, both pre- and post-development rents are nondecreasing over time:  $R'(t) \geq 0$ , and  $v'(t) \geq 0$  for any  $t$ . Second, the rent of developed land rises faster than that of undeveloped land, i.e.,  $R'(t) > v'(t)$  for any  $t$ . Third, the net land rent of developed land is lower than that of undeveloped land at the initial time but eventually becomes higher:  $r(0) < v(0)$  but  $r(\infty) > v(\infty)$ .

If renting were equivalent to owning, capital gains taxation would be irrelevant because the owner could then escape the tax altogether by renting the land to a potential buyer. We therefore assume that the current owner of the land lacks the ability to manage development of the land and that renting the land is inefficient compared with owning the land.<sup>2</sup>

The buyer of the land does not have to develop it immediately. We assume that the buyer can earn the same pre-development rent  $v(t)$  as the seller would have done until development.

In our model, the only choice is when to sell and develop the land. Let  $S$  denote the time when the landowner sells the land, and  $T$  the time when the buyer develops it. We assume that there are many potential buyers whose future earnings profiles are the same. Competition among buyers then raises the land price until it coincides with the discounted value of future earnings. The land price before tax is

$$q(S, T) = e^{-i(T-S)} \int_T^\infty r(t) e^{-i(t-T)} dt + \int_S^T v(t) e^{-i(t-S)} dt \quad (1)$$

at time  $S$ . Partial differentiation of equation (1) with respect to  $S$  yields

$$\frac{\partial q(S, T) / \partial S + v(S)}{q(S, T)} = i. \quad (2)$$

The left hand side is the rate of return on land which is the sum of capital gains and income gains. This must equal the rate of return on an alternative asset, i.e., the interest rate  $i$ .

The seller has to pay a tax on capital gains  $q(S, T) - p_0$ , where  $p_0$  is the basis for the tax. The basis is usually the price at which the seller acquired the land but the analysis in this paper does not require that it be an equilibrium price. The owner might have been lucky or unlucky in buying the land at a disequilibrium price; or the purchase price was formed under rational expectations but uncertainty was resolved after the purchase of land. One of our major results is that there will be no lock-in effect if there is no uncertainty and if the purchase price was formed under perfect foresight.

If the capital gains tax rate is  $\tau$ , the tax liability is  $\tau[q(S, T) - p_0]$ . We do not rule out the possibility that capital gains are negative. In such a case, the seller receives a subsidy from the government. Because land price rises over time in our model, however, capital gains are nonnegative unless the basis  $p_0$  is higher than the actual purchase price. The land price after tax is

$$p(S, T; \tau, p_0) = q(S, T) - \tau[q(S, T) - p_0] \quad (3)$$

at time  $S$ .

The land value at time 0 is the discounted value of this land price plus the discounted sum of pre-development rents that the owner earns until the sale of land:

$$P(S, T; \tau, p_0) = [(1 - \tau)q(S, T) + \tau p_0]e^{-iS} + \int_0^S v(t)e^{-it} dt. \quad (4)$$

Note that upper-case letter  $P$  in (4) indicates the value of land at time 0 and lower-case

letter  $p$  in (3) denotes the (after-tax) price at time  $S$ . We omit  $\tau$  and  $p_0$  from the initial land value function  $P(S, T; \tau, p_0)$  when this does not cause confusion.

When the landowner sells the land, he will choose the buyer whose offer price is the highest. This means that, given the time of sale  $S$ , the development time  $T$  is chosen to maximize  $q(S, T)$ . The landowner then chooses  $S$  to maximize the initial land value  $P(S, T)$ . It is however easier to solve the problem in the opposite order. We first solve for the optimal selling time  $S$  given an arbitrary development time  $T$ . It will be shown that the optimal selling time is either the initial time 0 or just before the development time  $T$ , depending on the level of the basis  $p_0$ . We then solve for the optimal development time for each of these cases. Comparing them yields the optimal solution.

### 3. Timing of Sale

We first solve for the optimal selling time  $S$  given an arbitrary development time  $T$ . The rent profile is determined uniquely by the development time, and a change in selling time changes the tax payment only. Mathematically, combining (1) and (4) yields the initial land value,

$$P(S, T; \tau, p_0) = \int_0^T v(t)e^{-it} dt + \int_T^\infty r(t)e^{-it} dt - \tau[q(S, T) - p_0]e^{-iS}. \quad (5)$$

Because the first two terms are independent of  $S$ , the optimum for  $S$  requires that the present value of the tax liability,  $\tau[q(S, T) - p_0]e^{-iS}$ , be minimized.

This has two immediate implications. First, if the tax rate is zero, then the landowner is indifferent to the timing of sale. Second, if the basis  $p_0$  is the purchase price at time 0 and the price is formed under perfect foresight, then the optimal timing of

sale is  $S = 0$ . The reason is simple. In this case the basis is  $p_0 = q(0, T)$ , which implies that the tax liability is zero at time 0. Because the before-tax land price  $q(S, T)$  rises over time under our assumptions, the tax liability is positive for positive  $S$ . Hence, the present value of the tax liability attains its minimum (i.e., zero) at  $S = 0$ .

**PROPOSITION 1.** *If the basis equals the perfect foresight price at the initial time 0, then it is optimal for the owner to sell the land immediately.*

It is commonly believed that a tax on realized capital gains has lock-in effects. This proposition shows however that capital gains taxation has the opposite “lock-out” effect under perfect foresight. One way of understanding this result is that a tax on capital gains introduces a penalty for speculation. Note that in our model the current owner is basically a speculator because he himself cannot develop the land. The capital gains tax can be saved if the person who develops the land buys the land directly at time 0 without going through a speculator.

In general, if the basis of the asset is high compared with the current price, the capital gains tax induces the owner to sell the land immediately, and if it is low, the owner postpones the sale until (just before) development. Let us now turn to the demonstration of this more general result.

Because of our assumption that the pre-development rent is non-decreasing over time, it is never optimal to sell at an intermediate time between 0 and  $T$ . This can be seen by checking the second order condition for an interior optimum. The first order condition for an interior maximum of (5) is

$$\frac{\partial P}{\partial S} = \tau[v(S) - ip_0]e^{-iS} = 0, \quad (6)$$

where we used relationship (2) to simplify the expression. This solution does not satisfy the second order condition since we have

$$\frac{\partial^2 P}{\partial S^2} = \tau v'(S)e^{-iS} > 0 \quad (7)$$

at this point. Hence, the optimal selling time is one of the corners, i.e., either  $S = 0$  or  $T$ . Comparing these two corner solutions immediately yields the optimal selling time.

**PROPOSITION 2.** *Define a weighted average of pre-development rents  $v(t)$  with weights being  $e^{-it}$  by*

$$\bar{v}(T) \equiv \frac{\int_0^T v(t)e^{-it} dt}{\int_0^T e^{-it} dt} = \frac{i \int_0^T v(t)e^{-it} dt}{1 - e^{-iT}}. \quad (8)$$

Let  $\phi(T) \equiv \bar{v}(T) / i$  denote the present value of the infinite stream of the weighted average. Then, the optimal timing of sale is  $S = 0$  if  $p_0 > \phi(T)$ . If  $p_0 < \phi(T)$ , the optimal timing is (just before) the development time,  $S = T$ .

**PROOF:**

The two corner solutions satisfy

$$P(0, T) = (1 - \tau) \int_T^\infty r(t) e^{-i(t-T)} dt + \int_0^T v(t) e^{-it} dt + \tau p_0$$

and

$$P(T, T) = (1 - \tau) \int_T^\infty r(t) e^{-i(t-T)} dt + \tau p_0 e^{-iT} + \int_0^T v(t) e^{-it} dt.$$

The difference between them is



$$P(T, T) - P(0, T) = \tau \int_0^T v(t) e^{-it} dt - p_0(1 - e^{-iT}).$$

This implies

$$P(0, T) \underset{<}{>} P(T, T) \text{ as } p_0 \underset{<}{>} \phi(T).$$

**Q.E.D.**

If the basis is lower than  $\phi(T)$ , the usual lock-in effect arises: the owner has an incentive to postpone the sale of land in order to reduce the tax payment. We can interpret this result as follows.

As noted before, the owner chooses the selling time  $S$  to minimize the present value of the tax payment is  $\tau[q(S, T) - p_0]e^{-iS}$ . If the current value of the tax payment,  $\tau[q(S, T) - p_0]$ , is constant, its present value decreases over time and the owner prefers to defer the realization of the tax indefinitely. The reason is that the owner receives an implicit interest subsidy by deferring the tax payment. Because of land price appreciation, however, the tax payment increases over time. If it increases at a rate faster than the interest rate, the owner wants to advance the realization of the tax.

If the basis  $p_0$  is zero, then the tax payment increases at the same rate as the land price. Now, from (2) the rate of return on land equals the interest rate. Since the rate of return on land includes income gains as well as capital gains, the rate of land price appreciation cannot exceed the interest rate.

If the basis is positive, the tax payment,  $\tau[q(S, T) - p_0]$ , increases at a faster rate than the land price,  $q(S, T)$ , because the basis is constant over time. If the basis is large enough, therefore, the tax payment may increase faster than the interest rate. Put it

differently, the fact that the current value of the basis is constant means that its present value declines over time. This tends to increase the present value of the tax payment. If this effect is strong enough, the present value of the tax payment increases over time. Proposition 2 above shows that this “lock-out” case occurs when the basis is higher than  $\phi(T)$ . If, for example, the basis  $p_0$  is positive and the pre-development rent  $v(t)$  is zero, then the owner chooses to sell the land immediately.

The following corollary obtains sufficient conditions for lock-in and lock-out effects.

**COROLLARY.** *If the basis price  $p_0$  is lower than the capitalized value of the land rent at time 0, i.e.,  $p_0 < v(0) / i$ , then it is optimal to postpone the sale of land until the development time  $T$ . If the basis is higher than the capitalized value of pre-development rent at time  $T$ , i.e.,  $p_0 > v(T) / i$ , then selling at the initial time 0 is optimal.*

**PROOF:**

From (8),  $\phi(T)$  satisfies  $v(0) / i \leq \phi(T) \leq v(T) / i$  and  $\phi'(T) \geq 0$ . The corollary follows immediately from Proposition 2. **Q.E.D.**

Our results differ markedly from those obtained in models of security trading, e.g., Constantinides [4].<sup>3</sup> The essence of the argument there can be summarized as follows.

Consider an asset that the current owner purchased at price  $p_0$ . We call this asset land but it may be a security. Its price is  $p_1 (> p_0)$  at time  $t_1$ . If the owner sells it at time  $t_1$ , the capital gains tax of  $\tau(p_1 - p_0)$  is levied. Alternatively, the owner can

postpone the sale until her death at time  $t_2$  when the price is  $p_2$ . The capital gains tax is then  $\tau(p_2 - p_0)$ . In the latter case the after-tax value of the asset at the time of death is

$$W_H = p_2 - \tau(p_2 - p_0).$$

In the former case, it is assumed that the owner buys an asset of the same type (i.e., another plot of land with the same rate of return) again at time  $t_1$ . The after-tax value of the land is then  $p_2 - \tau(p_2 - p_1)$  at the time of death. We must subtract from this the value of the tax paid at time 1. If the discount rate is  $i$ , the value of the tax at time 2 is  $\tau(p_1 - p_0)e^{i(t_2 - t_1)}$ , and the net worth of the owner is

$$W_S = p_2 - \tau(p_2 - p_1) - \tau(p_1 - p_0)e^{i(t_2 - t_1)}$$

at that time. Comparing this with  $W_H$  above yields

$$W_H - W_S = \tau(p_1 - p_0)[e^{i(t_2 - t_1)} - 1] > 0.$$

This inequality implies the presence of the lock-in effect: even if the new plot of land yields a higher rate of return, the owner is willing to hold the old one so long as the difference in returns is smaller than  $W_H - W_S$  above.

The results in this paper are based on a different comparison. The point of departure is the choice of the asset at time  $t_1$ . In the above comparison, the revenue from the sale of land at time  $t_1$  is invested in another plot of land. The use of discount rate  $i$ , however, means that the owner has an access to an asset whose net rate of return is  $i$ . When she sells the asset at time  $t_1$ , she can invest in this alternative asset and the net worth at time  $t_2$  is

$$W_B = [p_1 - \tau(p_1 - p_0)]e^{i(t_2 - t_1)}.$$

Now, suppose the rate of return on land (before the capital gains tax) equals  $i$ .

Then the lock-in effect means that  $W_H > W_B$ . We can easily see that this inequality may not hold. Comparing  $W_B$  with  $W_H$  yields

$$W_H - W_B = (1 - \tau)[p_1 e^{i(t_2 - t_1)} - p_2] - \tau p_0 [e^{i(t_2 - t_1)} - 1].$$

This is negative if the basis  $p_0$  is sufficiently high. An example of this is the case where the asset yields no income gains but the basis  $p_0$  is strictly positive.<sup>4</sup> If income gains (i.e., land rents) are zero, then the price of the land must rise at rate  $i$ . This makes the first square bracket zero, and  $W_H - W_B$  is negative if  $p_0$  is positive.

#### 4. Timing of Development

We have seen that the owner sells the land either at the initial time or at the development time. This section examines the optimal development time for each of these two cases.

If the owner sells the land at time 0, the capital gains tax will not distort the development time. This is obvious because the tax that is paid at time 0 does not influence the decisions after that.

If the owner sells the land at time  $T$ , the development time will be distorted. In this case the owner wants to postpone the realization of the capital gains tax until development, and postponing development allows further deferral of the capital gains tax.

##### The First Best Development Time

Let  $T^*$  denote the first best development time that is obtained when the tax rate is zero. Then, it is easy to see that  $T^*$  satisfies

$$r(T^*) = v(T^*), \quad (9)$$

where  $r(t) = R(t) - iC$  is the net post-development rent defined in section 2. This condition is well known in the literature on durable housing (surveyed in Fujita [5]).<sup>5</sup>

### Sale at the Initial Time

If the owner sells the land at time 0, the buyer chooses the development time  $T$  to maximize  $q(0, T)$ . Because the buyer does not pay the capital gains tax, the development time in this case coincides with the first best.

**PROPOSITION 3.** *If the owner sells the land at time 0, then the optimal development time coincides with the first best,  $T^*$ .*

### Sale at the Development Time

When the owner sells the land just before development, the value of land at the initial time is

$$P(T, T; \tau, p_0) = \{(1 - \tau) \int_T^\infty r(t) e^{-i(t-T)} dt + \tau p_0\} e^{-iT} + \int_0^T v(t) e^{-it} dt. \quad (10)$$

The development time is chosen to maximize this initial land value. Let  $T^{**}(\tau, p_0)$  denote the solution to this maximization problem:

$$T^{**}(\tau, p_0) = \arg \max_{\{T\}} P(T, T; \tau, p_0). \quad (11)$$

If  $T^{**}(\tau, p_0)$  is an interior solution, it satisfies

$$r(T^{**}(\tau, p_0)) - v(T^{**}(\tau, p_0)) = \tau [r(T^{**}(\tau, p_0)) - ip_0]. \quad (12)$$

This condition can be interpreted as follows. Deferring development changes the tax payment as well as the net rent earnings. The left hand side is the net loss of rent

earnings, and the right hand side is the value of tax savings. These two are equal at the interior solution.

### Optimal Timing for Sale and Development

So far we have seen that the development time is either  $T^*$  or  $T^{**}(\tau, p_0)$ . We next examine when these solutions are obtained. We first show that  $T^{**}(\tau, p_0)$  is optimal only when  $T^{**}(\tau, p_0) \geq T^*$ . That is, when the owner defers the sale of land until (just before) development, the development time cannot be earlier than the first best.

**PROPOSITION 4.** *If it is optimal for the owner to postpone the sale of land until (just before) development, then  $T^{**}(\tau, p_0) \geq T^*$ . If  $p_0 < \phi(T^*)$ , then the inequality is strict:  $T^{**}(\tau, p_0) > T^*$ .*

**Proof:**

Suppose the optimal development time  $T^{**}$  is strictly before  $T^*$ . Let  $\tilde{P}(T, \tau, p_0) \equiv P(T, T, \tau, p_0)$ . Then,  $T^{**}$  must satisfy the first order condition,

$$\frac{\partial \tilde{P}}{\partial T} = e^{-iT^{**}} \left\{ -[r(T^{**}) - v(T^{**})] + \tau[r(T^{**}) - ip_0] \right\} = 0.$$

We derive a contradiction by showing that  $\partial \tilde{P} / \partial T > 0$  at this point. From  $r(T^*) = v(T^*)$  and  $r'(T) > v'(T)$ , we have  $r(T^{**}) < v(T^{**})$ . Then, the above first order condition implies

$$-\tau[r(T^{**}) - ip_0] = -[r(T^{**}) - v(T^{**})] > 0.$$

Since the capital gains tax rate  $\tau$  must be less than 1 (one), we have

$$-\tau[r(T^{**}) - ip_0] < -[r(T^{**}) - ip_0].$$

Combining this inequality with the above equality yields

$$p_0 > v(T^{**}) / i,$$

which contradicts the corollary to Proposition 2 that requires  $p_0 \leq v(T^{**}) / i$ .

Next, if  $p_0 < \phi(T^*)$ , then the corollary to Proposition 2 implies that

$p_0 < v(T^*) / i$ . Since  $r(T^*) = v(T^*)$ , we obtain

$$\left. \frac{\partial \tilde{P}}{\partial T} \right|_{T=T^*} = e^{-iT^*} \left\{ \tau [v(T^*) - ip_0] \right\} > 0.$$

Hence, the optimal development time cannot be  $T^*$  if  $p_0 < \phi(T^*)$ .

**Q.E.D.**

This result is quite natural because the realization of the capital gains tax is linked to the development time in this case. Since the optimal selling time is a corner solution, gains from the deferral of the tax are not marginal. When there exist non-marginal tax advantages in postponing the sale, it also pays to postpone development since this allows longer deferral of the capital gains tax.

The next proposition obtains conditions under which the capital gains tax delays development.

**PROPOSITION 5.** Define  $\varphi(\tau)$  by  $P(T^{**}(\tau, \varphi(\tau)), T^{**}(\tau, \varphi(\tau)), \tau, \varphi(\tau)) \equiv P(0, T^*)$ ,

where  $\varphi(\tau)$  is unique and satisfies

$$\phi(T^*) \leq \varphi(\tau) \leq \phi(T^{**}(\tau, \varphi(\tau))).$$

Then, if  $p_0 \geq \varphi(T^{**}(\tau, p_0))$ , then the optimum is  $(S, T) = (0, T^*)$ . If

$p_0 \leq \varphi(T^{**}(\tau, p_0))$ , then the optimum is  $S = T = T^{**}(\tau, p_0)$ .

**PROOF:**

Let  $\Lambda(\tau, p_0) \equiv P(T^{**}(\tau, p_0), T^{**}(\tau, p_0), \tau, p_0) - P(0, T^*, \tau, p_0)$ . Then, using the envelope property, we obtain

$$\partial\Lambda/\partial p_0 = \tau e^{-iT^{**}} - \tau < 0.$$

Since  $\Lambda \geq 0$  at  $p_0 = \phi(T^*)$ ,  $\phi(\tau)$  which satisfies  $\Lambda(\tau, \phi(\tau)) = 0$  is unique, and

$$P(0, T^*) \begin{matrix} > \\ = \\ < \end{matrix} P(T^{**}, T^{**}) \text{ as } p_0 \begin{matrix} > \\ = \\ < \end{matrix} \phi(\tau).$$

Now Proposition 1 implies that equality  $P(0, T^{**}) = P(T^{**}, T^{**})$  holds at  $p_0 = \phi(T^{**})$ . Since  $P(0, T)$  achieves its maximum at  $T^*$ , we obtain

$$P(0, T^*) \geq P(0, T^{**}) = P(T^{**}, T^{**}) \text{ at } p_0 = \phi(T^{**}).$$

This implies  $\phi(T^{**}) \geq \phi(\tau)$ . Using similar arguments, we obtain

$$P(0, T^*) = P(T^*, T^*) \leq P(T^{**}, T^{**}) \text{ at } p_0 = \phi(T^*),$$

which implies that  $\phi(T^*) \leq \phi(\tau)$ .

**Q.E.D.**

Thus, if the basis for capital gains taxation  $p_0$  is higher than or equal to  $\phi(\tau)$ , then the lock-in effect does not arise. Instead, the owner sells the land at the initial time and the development timing coincides with the first best.<sup>6</sup> If the basis is lower than or equal to  $\phi(\tau)$ , then the owner postpones the sale until development. The development time in this case is not in general first best, and the distortion is always in the direction of delaying development.

In Proposition 1 we have already seen that, if the basis equals the perfect foresight price, the owner sells the land at time 0. The development time is then at the first best.



The lock-in effect therefore occurs only when there exist unexpected capital gains. This result is a special case of Proposition 5.

**COROLLARY 1.** *If the basis price is determined under perfect foresight, then the optimum is always  $(S, T) = (0, T^*)$ .*

Another implication of Proposition 5 is that, if the pre-development rent is zero, then there will be no distortion in the development time. If, in addition, the basis  $p_0$  is positive, then the owner chooses to sell at time 0. If  $p_0$  is zero, then the owner is indifferent among all points in the interval  $[0, T^*]$ . The last result corresponds to Proposition 8 in Sinn [8].

**COROLLARY 2.** *If  $v(t) = 0$  for any  $t$  and if  $p_0 > 0$ , then the optimum is  $(S, T) = (0, T^*)$ . If  $v(t) = 0$  for any  $t$  and if  $p_0 = 0$ , then all points in  $[0, T^*]$  are optimal for  $S$  and the optimal development time is  $T^*$ .*

A complete characterization of the solution is possible in a special case where the pre-development rent is constant and the post-development rent increases at a constant rate.

**COROLLARY 3.** *Suppose the post-development rent rises at a constant rate  $\theta$  and the pre-development rent is constant. Then, if  $p_0 > v / i$ , then the optimum is*

$(S, T) = (0, T^*)$ ; if  $p_0 < v / i$ , then the optimum is  $(S, T) = (T^{**}, T^{**})$ ; and if  $p_0 = v / i$ , then both  $(S, T) = (0, T^*)$  and  $(S, T) = (T^{**}, T^{**})$  are optimal, where

$$T^{**} = \frac{1}{\theta} \left[ \ln(v + iC) + \frac{\tau}{1-\tau} (v - ip_0) - \ln(R) \right] \quad (13)$$

is the interior solution that satisfies (12).

**PROOF:**

If  $R(t) = Re^{\theta t}$  and  $v(t) = v$ , then  $T^{**}(\tau, p_0)$  in (12) is unique and satisfies (13).

At  $T^{**}$  the second order condition for maximum is always satisfied

$$\partial^2 \tilde{P} / \partial T^2 = -(1-\tau)r'(T^{**})e^{-iT^{**}} < 0.$$

The uniqueness of  $T^{**}$  then implies that  $T^{**}$  maximizes  $\tilde{P}$  globally.

Next, it is easy to see that  $T^*$  satisfies

$$T^* = \frac{1}{\theta} \left[ \ln(v + iC) - \ln(R) \right].$$

Hence,  $T^{**} \geq T^*$  only if  $p_0 \leq v / i$ . From  $\phi(T) = v / i$ , Proposition 5 yields the corollary.

**Q.E.D.**

## 5. Concluding Discussions

We have seen that taxation of realized capital gains may or may not cause the lock-in effect. The lock-in effect occurs when the basis for taxation is low, but if the basis is high, the tax induces the owner to sell her land immediately. This reflects the fact that the owner who cannot develop the land by herself is equivalent to being a

speculator. Because the capital gains tax represents an additional cost for speculative trading, it discourages speculative holding of land.

The result that the lock-in effect may or may not arise depending on the the level of the basis is not limited to land development. The same result obviously holds for securities. Distortions in real resource allocation are however different in the case of securities. In a public corporation where equity owners are separate from the management, whether or not equity owners are locked in does not have any direct effects on the management of the corporation. Distortionary effects arise only through distortions in the portfolio of locked-in owners.

## References

- [1] Auerbach, A. J., 1991, Retrospective capital gains taxation, *American Economic Review*, 81, 167-178.
- [2] Becker, G., Investment in human capital: a theoretical analysis, *Journal of Political Economy*, 70, Suppl., 9-49 (1962).
- [3] Bentick, B. L., The impact of taxation and valuation practices on the timing and efficiency of land use, *Journal of Political Economy*, 87, 859-874 (1979).
- [4] Constantinides, G. M., Capital market equilibrium with personal tax, *Econometrica*, 51, 611-636 (1983).
- [5] Fujita, M., Urban land use theory, in "Location Theory" (Fundamentals of Pure and Applied Economics, Vol. 5), 73-149 (1986).
- [6] Kanemoto, Y., Housing as an asset and the effects of property taxation on the residential development process, *Journal of Urban Economics*, 17, 145-166 (1985).

- [7] Kanemoto, Y., Contract types in the property market, *Regional Science and Urban Economics*, 20, 5-22 (1990).
- [8] Sinn, H-W., Vacant land and the role of government intervention, *Regional Science and Urban Economics*, 16, 353-385 (1986).
- [9] Skouras, A., The non-neutrality of land taxation, *Public Finance*, 33, 113-134 (1978).
- [10] Wildasin, D., More on the neutrality of land taxation, *National Tax Journal*, 35, 105-111 (1982).

## Footnotes

<sup>1</sup> An earlier version of this paper was presented at the meetings of the Japan Association of Economic Theory and Econometrics Association. I thank Tatsuo Hatta for useful discussions. I also thank two referees and Jan Brueckner for insightful comments that improved the exposition of the paper.

<sup>2</sup> Kanemoto [7] shows that the lack of verifiability makes a rental contract of land inferior to the sale of land.

<sup>3</sup> Auerbach [1] which presents a method of capital gains taxation that eliminates the lock-in effect also contains a similar explanation of the lock-in effect.

<sup>4</sup> Note that, even if income gains are zero in the current period, the asset price can be positive so long as investors expect future income gains.

<sup>5</sup> Extensions of this condition appear in the literature on distortionary effects of land value taxation, e.g., Bentick [3], Skouras [9], and Wildasin [10].

<sup>6</sup> This result differs from those in Kanemoto [6] which showed that capital gains taxation slows down land development. The difference is caused by our implicit assumption on development technology. This paper assumes indivisibility in development so that the only choice variable is the timing of development. Kanemoto [6] assumes that the unit cost of housing capital does not depend on the size of development. Under this assumption, development occurs continuously and the choice variable is how much land to develop at each instant of time.