APPENDIX I

EQUALITY AND THE BENTHAMITE SOCIAL WELFARE FUNCTION

In this appendix we explain the reason why utility levels differ between different locations at the Benthamite optimum. The basic reason is that the utility possibility frontier is skewed in favour of households living farther from the center. This can be illustrated by considering a rectangular city consisting only of two households. For notational simplicity, the width of the city is assumed to be 1, i.e., $q(x)=1$. Household i consumes z_i of the consumer good and h_i of space. Both households are assumed to commute to the center of the city from the center of their properties, i.e., commuting costs of household 1 and 2 are respectively $t(\frac{1}{2}h_1)$ and $t(h_1+\frac{1}{2}h_2)$, where household 1 lives closer to the center. This city is illustrated in Figure 1.

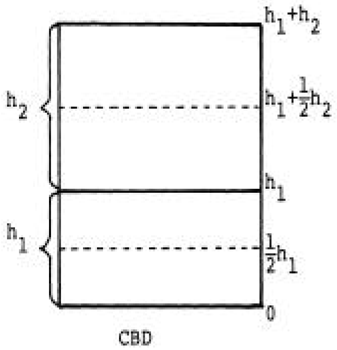

Figure 1. A Rectangular Two-Household City

The resource constraint for the city is given by

$$z_1+z_2+t(\frac{1}{2}h_1)+t(h_1+\frac{1}{2}h_2)+R_a(h_1+h_2)=Y. \quad (1)$$

Given the resource constraint, we can obtain the set of feasible utility levels of the two households. The frontier of the set is called the *utility possibility frontier* and depicted by the curve LL' in Figure 2. The utility possibility frontier expresses the maximum utility that household 1 can achieve at every possible utility level for household 2. It is obtained by maximizing u_1, subject to the resource constraint and to

$$u(z_2, h_2) \geq u_2. \tag{2}$$

The Lagrangian is therefore

$$\begin{aligned}\Lambda = u(z_1, h_1) + d\left[Y - z_1 - z_2 - t(\frac{1}{2}h_1) - t(h_1 + \frac{1}{2}h_2) - R_a(h_1 + h_2)\right] \\ + l\left[u(z_2, h_2) - u_2\right].\end{aligned} \tag{3}$$

The first order conditions can be summarized as

$$l = \frac{u_z(z_1, h_1)}{u_z(z_2, h_2)} = \frac{u_h(z_1, h_1)}{u_h(z_2, h_2)} - \frac{1}{2}d\,\frac{t'(\frac{1}{2}h_1) + t'(h_1 + \frac{1}{2}h_2)}{u_h(z_2, h_2)}. \tag{4}$$

Now it can be shown that the utility possibility frontier; is skewed as in Figure 2, so that its slope is flatter than minus 1 when the two households obtain the same utility level.

By the Envelope Theorem in Appendix III, the slope of the utility possibility frontier is

$$\frac{du_1}{du_2} = \frac{\partial \Lambda}{\partial u_2} = -l = -\frac{u_z(z_1, h_1)}{u_z(z_2, h_2)}. \tag{5}$$

Since the last term on the RHS of (4) is negative, we have

$$\frac{u_h(z_1, h_1)}{u_z(z_1, h_1)} > \frac{u_h(z_2, h_2)}{u_z(z_2, h_2)}. \tag{6}$$

Thus the slope of an indifference curve is steeper at (z_1, h_1) than at (z_2, h_2). Due to the convexity of indifference curves, this implies, as shown in Figure 3, that $z_1 > z_2$

and $h_1 > h_2$ if $u_1 = u_2$. But if land is a normal good, the following inequality is obtained from (I.2.7) and (I.2.8):

$$\begin{aligned} \frac{du_z(z,h)}{dz}\bigg|_{u=const.} &= u_{zz} + u_{zh}\frac{dh}{dz}\bigg|_{u=const.} \\ &= u_{zz} - u_{zh}\frac{u_z}{u_h} \\ &= -D(u_h/u_z)\hat{h}_I < 0. \end{aligned}$$

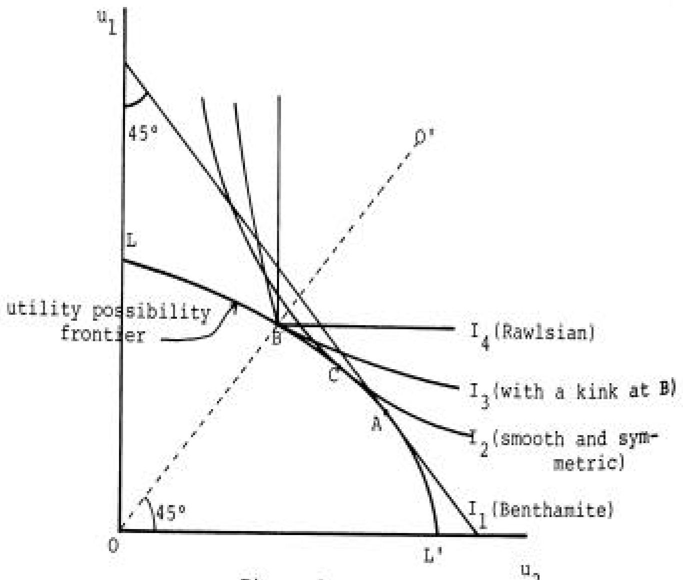

Figure 2

Hence, u_z decreases as z increases along an indifference curve and we finally obtain

$$\frac{du_1}{du_2}\bigg|_{u_1=u_2} = -\frac{u_z(z_1,h_1)}{u_z(z_2,h_2)} > -1.$$

As the simple sum of utilities is maximized in the Benthamite case, the Benthamite optimum is point A in Figure 2 at which the 45 ° line I_1 is tangent to the utility possibility frontier. Since the utility possibility frontier is flatter than 45 ° when utility levels are equal, the optimum must lie below the equal-utility line 00'.[1] Thus household 2 receives a higher utility level than household 1.

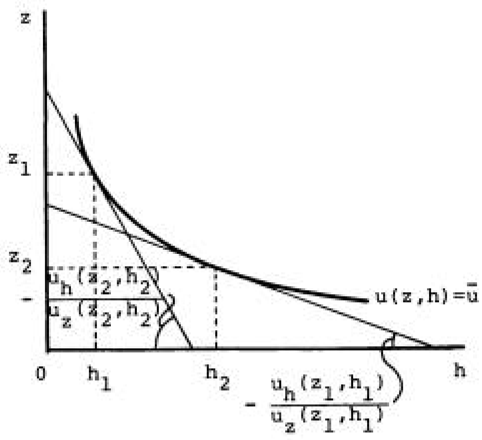

Figure 3

This result generalizes to any *smooth* symmetric quasiconcave social welfare function, $W(u_1,u_2)$, represented by indifference curves like I_2 since all indifference curves of a smooth symmetric social welfare function must have slope -1 along the equal-utility line 00'. A symmetric quasi-concave social welfare function yields equal utility levels only if indifference curves have kinks along the 45 ° line, as I_3 does. One example is the Rawlsian case represented by I_4.

The skewed utility possibility frontier is a result of the so-called concealed nonconvexity. In our model, it is assumed that a household must choose only one location and cannot live at more than one location at a time. This assumption can be interpreted in two ways. First, it may be considered as a restriction on the consumption set. For example, in Figure 4, which describes housing consumptions at two locations x and x', the consumption set is limited to the two axes, and any point within the first quadrangle cannot be chosen. In this case, the consumption set is not obviously convex. Second, the assumption may be a consequence of nonconvex preferences. If indifference curves are concave to the origin as in the Figure 4, a household, given a linear budget constraint, always chooses one of the corners.

[1] It is implicitly assumed that the utility possibility set is convex. This is true if the transportation cost function is convex and the utility function is concave.

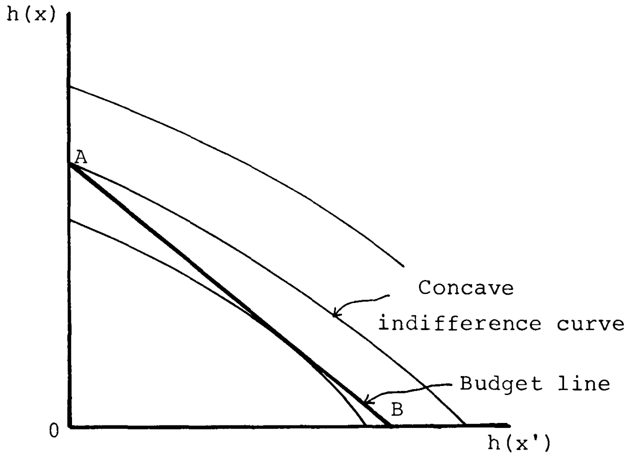

Figure 4. Concealed Nonconvexity

In our model, this nonconvexity is not harmful for the existence and efficiency of competitive equilibrium, since enough smoothness is obtained by introducing a population density function. The crucial assumption is that the population density at each distance can be any real number. This is not true in a model with several regions in which the population in each region must be an integer. In such a case demand for land in one region is discontinuous at the price level where an individual moves in or out of the region. Hence, there may never be a price vector that equilibriate the market for land. If, however, the population in a region can be any real number, such discontinuity will not occur and the existence of competitive equilibrium will be guaranteed. Schweizer, Varaiya and Hartwick (1976) proved that competitive equilibrium exists in a model with the concealed nonconvexity if a population density can be any real number.

This result is analogous to the well-known result in general equilibrium theory (due to Star (1969) and others) that nonexistence of equilibrium caused by nonconvexities of individual units disappears as the economy becomes larger relative to individual economic units. In particular, it is parallel to the work of Aumann (1966) which shows that in a model with a continuum of households, each of infinitesimal endowment, the existence of competitive equilibrium can be proven without making any assumption about convexity.

Although the nonconvexity does not introduce any difficulty concerning the existence and efficiency of competitive equilibrium, it causes inequality in utility levels. The asymmetry in the utility possibility frontier arises since housed holds living near the center, for example, are not allowed an access to land in the suburbs. In such a case, households living at different locations face different opportunity sets. If, however, households can live at more than one location, they all face exactly the same budget constraint and there is no difference between households. The utility possibility frontier is then symmetric and all households receive the same utility level at the Benthamite optimum.

Some economists prefer the Benthamite case on the grounds that the Rawlsian social welfare function must be assumed to obtain the equal-utility optimum. As can be seen from Fig. 2 however, this claim is not true. Utility levels are equal at the optimum if the social welfare indifference curves have sufficiently strong kinks along the equal-utility line.

Any symmetric indifference curve with no kinks has a slope -1 along the equal utility line. This implies that in the neighborhood of the equal utility line the social welfare function behaves in the same way as the Benthamite social welfare function. Thus at least locally the aggregate utility is maximized and the social welfare function exhibits no preference for equality of utility levels. If local preference for equality is assumed at the point where utility levels are equal, indifference curves will have kinks and utility levels may be equal at the optimum.

REFERENCES

Aumann, R.J., (1966), "Existence of Competitive Equilibria in Markets with a Continuum of Traders," *Econometrica* 34, 1-17.

Schweizer, U., P. Varaiya, and J. Hartwick, (1976), "General Equilibrium and Location Theory," *Journal of Urban Economics* 3, 285-303.

Starr, R., (1969), "Quasi-equilibria in Markets with Nonconvex Preferences," *Econometrica* 37, 25-38.