CHAPTER III
LOCAL PUBLIC GOODS

The spatial equilibrium model in Chapter I can be used to analyze problems associated with optimal provision of local public services. In the case of *pure* public goods it is extremely difficult to achieve the optimal allocation by a decentralized mechanism. Local public goods which, while still public, are not perfectly public, however, allow the introduction of competition among suppliers and it is possible to devise a competitive mechanism which achieves the optimal allocation.

A pure public good is consumed collectively: its consumption by any individual does not reduce the amount available for others. The classic example of a virtually pure public good is national defense. It is claimed that the amount of "security" one person "consumes" from her nation's "defense expenditure" has no effect on the amount available for others: the entire population is able to consume a pure public good.

Conventional public good theory assumes that the number of consumers is fixed since the size of the community - usually a nation - is known. For local public services the assumption breaks down because the population of local communities is endogenous, determined in the system's search for equilibrium.[1] It is possible to take advantage of this problem, however.

We know from the Fundamental Theorem of Welfare Economics that, if there are only pure private goods, a competitive equilibrium is Pareto optimal, that is, no one can be made better off without making somebody else worse off. When there are public goods, however, a competitive equilibrium fails to attain Pareto optimality, and furthermore it is difficult to devise any other workable decentralized mechanism. The problem arises because households have an incentive to "misreveal" their preferences. By understating the marginal benefit it gains from the public good, a household can avoid being assessed its full share of the cost of providing the good, without suffering a reduction in supply. Supply is unaffected , because the contribution of a single household is negligible. This difficulty is often called the "free rider" problem.

Since a pure public good is consumed by all households concurrently, a marginal increase in supply benefits all households simultaneously. The marginal social benefit is therefore the sum of the benefits received by each household, which may be expressed as the sum of marginal rates of substitution between the public good and the numeraire.

If all households were to pay the full value of the benefit they received, profit maximization would yield an efficient allocation. Because of the free rider problem, however, it is extremely difficult to devise a pricing scheme in which every household has an incentive to reveal its marginal evaluation of the public good.[2]

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[1] Stiglitz (1977) emphasized this aspect of local public goods.

[2] Although dark (1971), Groves (1973) and Groves and Ledyard (1977) invented a mechanism in which a household has an incentive to reveal its preferences correctly, this mechanism is rather artificial. Green and Laffont (1977) proved that this mechanism is the only one that does not have the preference revelation problem.

In the case of local public goods, competition between different communities can work in a manner similar to competition between suppliers of private goods. The preference revelation problem still remains *within* a community since a local public good has the same characteristics as a pure public good within a community. It is, however, possible to exploit the special property of local public goods, the fact that the population of beneficiaries is endogenous to the system. If a community increases the supply of local public goods, the community becomes more attractive, which induces immigration of households. This increases demand for housing, causing land rent to rise. The marginal social benefits of the public goods are therefore reflected, at least partially, in the marginal increase in land rent. If the community is infinitely small relative to the rest of the world, the marginal benefits equal the increase in the total land rent in the community. Then the behaviour which is characteristic of a land developer, maximizing land rent net of the cost of providing the public goods, leads to the efficient supply of the public goods.

In order to illustrate the basic principle, we start in section 1 with a simple case. Public goods are assumed to be extremely local in the sense that they are jointly consumed only by residents at a location. To simplify the analysis we assume that public goods supplied at a certain distance from the city center can be consumed only by residents living at that distance from the center. In effect we pretend that neighbourhoods form a series of concentric rings, each of unit width, around the city center. It may seem a bit peculiar, but the assumption is nothing more than a mathematical convenience which yields perfectly sensible and general results. This type of public good represents, in an extreme form, goods consumed only by households living very close to the location of supply; street lighting, for example, or neighbourhood beautification, or snow removal. The extremely local public goods are embedded in the closed city of Chapter I.

Not surprisingly, the optimum solution must meet the Samuelsonian condition that the sum of marginal rates of substitution be equal to the marginal cost of the public good. Another interesting property of the optimal solution is that the differential rent (the difference between the urban rent and the rural rent) at the edge of the city equals the cost of the public good there.

The optimal solution can be achieved either by centralized control, which requires impractical amounts of information, or through a decentralized mechanism such as a system of neighbourhood development corporations which rent land at the rural rent and maximize their profits. In the second half of section 1 a system of competitive land developers with a developer in each neighbourhood is described and its optimality demonstrated.

In section 2 we examine a crowding phenomenon by assuming that the cost of producing the same amount of the public good rises as the number of residents increases. The major difference in this case is that the optimal solution requires a congestion tax on households. The congestion tax at any location equals the marginal increase in the cost of the public good caused by adding a household there. The system of competitive land developers achieves the optimal allocation if a land developer charges the congestion tax and maximizes rent plus tax minus the costs of providing the public good.

In section 3 we consider a local public good which is jointly consumed by all residents in an entire city, rather than by residents at a certain radius. Museums, theaters, sewage systems, and large parks may fit this category. Competition between cities is introduced by assuming that there are many identical cities. The results are parallel to those in the increasing-returns-to-scale case of the previous chapter, as well as those in the extremely local public good case of the present chapter. If a competitive land developer develops an entire city, the local public good is optimally supplied when the number of cities is very large. Moreover, the zero profit condition from free entry insures the optimum number of cities.

Crucial in deriving our results is that, in the eyes of a developer, the utility level of the residents is fixed. This suggests that we can extend the result to more general models as long as this condition is guaranteed. In Appendix II we consider one example of such an extension, in which two inputs, land and labour, are used in production.

It is worth emphasizing that our results depend on the assumption that all households in the economy are identical in terms of both skills and preferences. Although we may relax this assumption to include different types of households, we must assume that there are many households in each type in the whole economy and that one region contains a very small fraction of the households in each type. Since identical households receive the same utility level in equilibrium, regardless of where they live, a change in the supply of local public goods in one small region has a negligible effect on the general utility level. If all households are different, however, the utility levels of residents cannot be taken as constant even in the case where the population of the region is very small compared with the rest of the world. Therefore, at best we can only say that the system of competitive land developers approximates the optimal allocation of local public goods. How good an approximation it achieves is an empirical question. Considering the fact that there is no perfect mechanism to supply public goods, however, our scheme of letting competitive land developers supply local public goods is worth a serious consideration. Our result would suggest, for example, that when a land developer develops a new community, the developer rather than a local government should pay for the public good supplied in the community.

1. An Extremely Local Public Good

Consider an extremely local public good in the public-ownership, closed-city case of Chapter I. The amount of public good supplied between x and $x+dx$ is denoted by $X(x)dx$. Though we consider only one public good for notational simplicity, the conclusions obtained in this section are valid for any number. The public good is extremely local in the sense that the public good supplied at x is jointly consumed only by residents of a ring of unit width between $x-\frac{1}{2}$ and $x+\frac{1}{2}$. If we assume that public goods supplied at different radii are perfect substitutes, then a household at x had available.

$$\int_{x-1/2}^{x+1/2} X(x')dx'.$$

or approximately $X(x)$, of the public good and its utility function can be written

$$u(z(x), h(x), X(x)). \tag{1.1}$$

It is assumed that the consumer good is the only input in the production of the public good. The public good is assumed to be produced separately at each location at a cost $C(X(x))$. Then the resource constraint (I.1.30) is rewritten as follows.

$$\int_0^{\bar{x}} \{[z(x)+t(x)]N(x)+c(X(x))+R_a q(x)\}dx = F(P) \tag{1.2}$$

The land constraint is the same as (I.2.2), and the population constraint as (1.1.24):

$$q(x) = h(x)N(x) \tag{1.3}$$

$$P = \int_0^{\bar{x}} N(x)dx\ . \tag{1.4}$$

The sum of the equal utilities,

$$\int_0^{\bar{x}} N(x)dx\ , \tag{1.5}$$

is maximized under the constraints (1.2), (1.3), (1.4) and the equal utility constraint,

$$u(z(x), h(x), X(x)) = u\ . \tag{1.6}$$

The first order conditions for this problem become, after simple rearrangements:

$$\frac{u_h}{u_z} = R(x)\ , \tag{1.7a}$$

$$\frac{u_X}{u_z} N(x) = c'(X(x))\ , \tag{1.7b}$$

$$y = z(x) + t(x) + R(x)h(x)\ , \tag{1.7c}$$

$$R(\bar{x}) - R_a = \frac{c(X(\bar{x}))}{q(\bar{x})}\ . \tag{1.7d}$$

(1.7a) and (1.7c) are the same as in Chapter I. (1.7a) equates the marginal rate of substitution between housing and the consumer good to the shadow rent. (1.7c) states that the household expenditure on private goods, evaluated at the shadow prices, must be the same everywhere in the city.

Conditions (1.7b) and (1.7d) are new. (1.7b) is the Samuelsonian condition for efficient supply of the public good described in the introduction: the marginal cost of the public good at x must equal the sum over all residents at x of the residents' marginal rates of substitution between the public good and the consumer good. A unit increase in the supply of the public good between x and $x + dx$ raises the utility level of a household there by u_X. Since $N(x)dx$ households receive the benefits of the public good, the marginal social benefit in utility terms is $u_X N(x)dx$, and in pecuniary terms $(u_X / u_Z)N(x)dx$. The social optimum is achieved when the marginal benefit equals the social marginal cost, $c'(X(x))dx$.[3]

(1.7d) shows that the shadow rent at the boundary of the city is not equal to the rural

[3] If we go back to the original formulation, a household at x has available $\int_{x-\frac{1}{2}}^{x+\frac{1}{2}} X(x')dx'$ of the public good. Consider the costs and benefits of a unit increase of $X(x)$ between x and $x+dx$. The costs are $c'(X)dx$. On the other hand, the utility level of a household between $x-1/2$ and $x+1/2$ rises by $u_X dx$, and the marginal benefit a household receives is $(u_X / u_z)dx$ in pecuniary terms. The social benefit is obtained by summing this over all households between $x-1/2$ and $x+1/2$ so that the optimality condition is

$$\left[\int_{x-\frac{1}{2}}^{x+\frac{1}{2}} \frac{u_X}{u_z} N(x')dx'\right]dx = c'(X(x))dx\ .$$

Equation (1.7b) is obtained if we can approximate $(u_X / u_z)N(x')$ for all x' between $x-1/2$ and $x+1/2$ by $(u_X / u_z)N(x)$.

rent as in Chapter I, but rather greater than the rural rent by the cost-per-unit-area of producing the public good there.

This optimal solution can be achieved in the following ways. First, local governments might supply the local public good so as to equate the sum of marginal rates of substitution to marginal cost of the public good at each location. The city would lease the land to those who pay the highest rent, which would be $R(x) = u_h / u_Z$ in market equilibrium. Part of the revenue would then be used to produce the public good and the rest returned to residents as an equal subsidy. The public good would be supplied out to the radius where the market rent minus the rural rent equals the cost of the public good per unit acre. Under this arrangement utility maximization by households ensures conditions (1.6) and (1.7a) and the market equilibrium attains the optimal allocation.[4] Unfortunately, this method is not practical since local governments must know the marginal rates of substitution, and these are very hard to discover.

The second way to implement the optimal solution can be seen as a system of land developers. Imagine a large number of developers in a city, each developing an extremely small area, and each supplying the local public good in their area. The developers rent land from the rural landlords and sublet it to city residents at the market rent. In our circular city, it is convenient to allow each developer to develop a band around the city center at a given radius. The developer's profit, which is the differential rent minus the cost of providing the public good, becomes

$$\left[R(x) - R_{\bar{a}}\right] q(x) - c(X(x)) \,.$$

In order to ensure that all households obtain the same utility level, we assume that the profit is distributed equally among all city residents.

Since each developer is very small, its action does not significantly affect the utility or the income levels. Therefore, when he changes the supply of the public good, land rent moves in such a way that utility and income both remain unchanged. The change in land rent can be obtained as follows. A household maximizes the utility function (1.1) under the budget constraint (1.7c), which can be summarized as the indirect utility function,

$$v(y - t(x), R(x), X(x)) \qquad (1.8)$$

as in (I.1.7). Equating the indirect utility function to the fixed utility level, u, we obtain the

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[4] The reader may wonder whether a household would not prefer to rent land directly from the rural owners or the central government and live outside the boundary of the city, where the public good is not supplied. If the optimal solution requires a positive supply of the public good at the boundary of the city, then households do not have an incentive to live in the places where the public good is not supplied. It suffices to show that households obtain higher utility at the boundary if the public good is supplied than not, since locations farther than the boundary are even less desirable.

From (1.7c) and (1.7d), the following resource constraint is satisfied at $\bar{x}$.

$$y = z(\bar{x}) + t(\bar{x}) + R_a h(\bar{x}) + \frac{c(X(\bar{x}))}{N(\bar{x})} \qquad (*)$$

A household which lives on the other side of $\bar{x}$ has the budget constraint;

$$y = z + t(\bar{x}) + R_a h \,. \qquad (**)$$

Since the same amount of resource is used up in both cases, under (*) be higher than or equal to the maximum attainable utility level under the budget constraint (**). Otherwise, the utility level of $\bar{x}$ can be increased by making the supply of the public good zero without lowering the utility level of other locations.

bid rent function,

$$R(y - t(x), u, X(x)) \tag{1.9}$$

as in (I.1.12).

A profit maximizing developer at x maximizes

$$[R(y - t(x), u, X(x)) - R_a]q(x) - c(X(x)) \tag{1.10}$$

with respect to $X(x)$, which yields

$$R_X q(x) = c'(X). \tag{1.11}$$

This implies that the optimality condition (1.7b) is satisfied. By Roy's Identity (1.1.10) the bid rent function satisfies

$$R_X = \frac{l}{h}\frac{v_X}{v_I}.$$

Noting that $v_X = u_X$ and $v_I = u_Z$ by the Envelope Theorem[5], we can rewrite this equation as

$$R_X = \frac{l}{h}\frac{u_X}{u_Z}. \tag{1.12}$$

Equation (1.5b) follows, since from the land constraint (1.3), $q(x)/h(x) = N(x)$.

The land developer operates only when profit can be made:

$$[R(x) - R_a]q(x) - c(X(x)) \geq 0. \tag{1.13}$$

This condition insures that (1.7d) is satisfied at the edge of the city.

Thus the system of land developers achieves the optimality conditions (1.7b) and (1.7d). Since other conditions are also satisfied in market equilibrium, the optimal allocation can be reproduced if the local public good is supplied by extremely small land developers.

Note that developers need to know only the land rent, and not the utility function. Therefore, the informational requirement is the same as the usual price mechanism. There still remains, however, a difference from the market system for private goods. Since firms and households maximize their objective functions taking prices as given, maximization processes are not affected by situations outside them, whereas the maximization problem for land developers involves an important *endogenous price,* namely, land rent, which is determined through reactions of households to the supply of the public good. Therefore, the profit-maximizing level of the , local public good can only be found after observing

[5] See Appendix III on the envelope property.

levels of land rent corresponding to many different supply levels.

The system of land developers may be interpreted as the mechanism proposed by Negishi (1972) and combining Margolis' principle of fiscal profitability with Tiebout's voting with one's feet. According to the principle of fiscal profitability, a local government pays for the local public good from a tax on land, and determines the supply of the public good which maximizes the rent net of the tax. This behaviour is identical to the profit-maximizing behaviour of a developer. Voting with one's feet allows households to choose the local government that offers the preferred bundle of local public goods. In our model the free choice of location represents voting with one's feet. This, coupled with the assumption of extremely small local governments, will insure that local governments take as given the utility level of residents.

The above result relies on the fact that the marginal benefits of the public good are capitalized in land rent. Multiplying (1.12) by $q(x)$, we obtain

$$q(x)R_X = \frac{u_X}{u_z}N(x): \tag{1.14}$$

the marginal increase in land rent at x, caused by a unit increase of the public good, equals the sum of the marginal rates of substitution between the public good and the consumer good, which in turn equals the marginal benefits of the public good. This result is characteristic of a small economy in which the utility level can be taken as given, and is independent of the public good being optimally supplied. The benefit of the public good must accrue to somebody or become a deadweight loss. Since there is no deadweight loss in the first best world, all the benefits must be received by somebody. In our model, the only place the benefits can go is land rent.

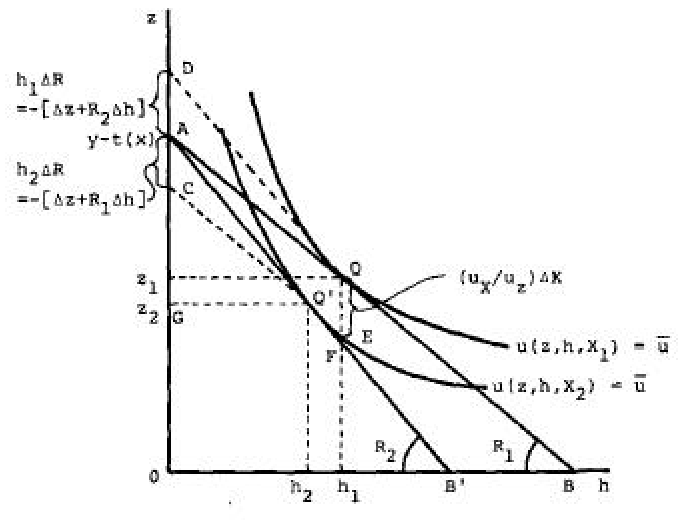

Figure 1. Capitalization of the Benefits of a Public Good

Figure 1 illustrates the capitalization of the benefits of public goods. Consider an increase in the supply of the public good from x_1 to x_2. Then a smaller bundle of (z, h) is necessary to achieve the same utility level, u, and the indifference curve shifts toward the origin. The equilibrium consumption moves from Q to Q'. The benefits of the increase in the supply of the public good can be represented by the amount of resources freed by this move. Since both z and h change, we must evaluate the change by using some relative price. There are at least two possibilities. If we use the before-the-change rent, R_1, the benefits of this change are AC in Figure 1, or $-[\Delta z + R_1 \Delta h]$; and if we use the after-the-change rent, R_2, the benefits are AD, or $-[\Delta z + R_2 \Delta h]$.

From Figure 1 (or from simple algebraic manipulations) it is clear that AC, which is $AG - CG$, also equals the change in rent, $\Delta R = R_1 - R_2$, multiplied by the after-the-change consumption level of housing, h_2, i.e., $h_2 \Delta R$; and that AD equals the change in rent multiplied by the before-the-change consumption, $h_1 \Delta R$.

Although it is not clear in this partial analysis which measure of benefit is a better approximation[6], if the change in X is infinitesimal, the two measures coincide, and the problem disappears. For a marginal change in X, therefore, the benefits a household receives equal $h \dfrac{dR}{dX}$, which is equivalent to (1.10). The social benefit is the sum of the benefits of all households who consume the public good and is given by $q(x) \dfrac{dR}{dX}$ in our model. Thus the rise in land rent completely capitalizes the marginal benefits of the public good.

The diagram also shows that the marginal rate of substitution between the public good and the consumer good is the correct measure of the marginal benefit of the public good which a household receives. When the consumption of land is held constant, a reduction in the consumption of the consumer good made possible by the increase in the public good equals QE. If the change in the supply of the public good is small, QE is approximately $\Delta z = (u_X / u_z) \Delta X$, since by total differentiation

$$u_z dz + u_X dx = du = 0,$$

where $\Delta X \equiv X_2 - X_1$. Moreover, as ΔX approaches zero, QF approaches $(u_X / u_z) \Delta X$. QF equals AD, and hence gives the benefit of the marginal increase evaluated at the after-the-change price. Thus u_X / u_z is the correct measure of the marginal benefit of the public good.

2. An Extremely Local Congestible Public Good

In the previous section we assumed that the local public good was a pure public good at each radius. In particular, we assumed that the costs of providing the same level of the public good did not depend on the number of consumers. This assumption does not hold for most public services. For example, the same park gives different levels of services depending on the number of people using it. The cost of providing the same level of park services usually increases as the number of users increases.

In this section we assume that the cost of producing the same level of the public good increases as population density increases. The cost function for the local public good is modified as

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[6] Following the approach due to Negishi (1972), Harris (1978) showed, in the context of public inputs rather than public consumption goods, that the value of the change evaluated at the after-the-change prices is the lower bound of the benefits and that the value at the before-the-change prices is the upper bound. Since in our case the value of the change evaluated at the after-the-change price is greater (in the absolute value) than the value at the before-the-change prices, Harris' result must clearly be modified. It is still an open question whether a similar relationship can be established in our model.

$$c(X(x), N(x)), \tag{2.1}$$

where

$$C_N > 0 \quad .$$

As in the previous section, the optimal solution can be easily obtained. The first order conditions are (1.7a) and

$$\frac{u_X}{u_Z} N(x) = C_{X'} \tag{2.2a}$$

$$y = z(x) + t(x) + R(x)h(x) + C_N \tag{2.2b}$$

$$c(X(\bar{x}), N(\bar{x})) = [R(\bar{x}) - R_a]q(\bar{x}) + N(\bar{x})c_N . \tag{2.2c}$$

(2.2a) is the same as before: the marginal cost of the public good must equal the sum of marginal rates of substitution between the public good and the consumer good for all households at each radius. Terms in (2.2b) and (2.2c) containing C_N are new. In order to achieve this solution in a market system, a household must pay a congestion tax equal to the marginal cost of adding a household, $c_{N'}$, and varying with distance from the center. Then (2.2c) states that the government budget is balanced at the edge of the city. The sum of the revenues from the congestion tax and the land rent is exactly equal to the sum of the rural rent and the cost of the public good at $\bar{x}$.

Consider again a system of competitive neighbourhood developers supplying the public good. As before we assume that no developer is large enough to affect the utility and income levels. We now assume that each developer charges a congestion tax (or the membership fee to join the location) and maximizes profit including the tax. If the congestion tax at x is denoted by $s(x)$, the developer at x maximizes

$$R(x)q(x) + s(x)N(x) - c(X(x), N(x)) . \tag{2.3}$$

The policy variables for the developer are $s(x)$ and $X(x)$. $R(x)$ and $N(x)$ are determined through the market's adjustment.

As in the previous section (c.f., Equation (1.9)), we can derive the bid rent function;

$$R(y - t(x) - s(x), u, X(x)) . \tag{2.4}$$

As in (I.1.14), the function satisfies

$$R_I(I(x), u, X(x)) = \frac{1}{h(x)}, \tag{2.5}$$

where $I(x) \equiv y - t(x) - s(x)$. The number of households at x, therefore, satisfies

$$N(x) = q(x)R_I(y - t(x) - s(x), u, X(x)) . \tag{2.6}$$

Thus a developer maximizes

$$\begin{aligned} &R(y - t(x) - s(x), u, X(x))q(x) \\ &+ s(x)q(x)R_I(y - t(x) - s(x), u, X(x)) \\ &- c[X(x), q(x)R_I(y - t(x) - s(x), u, X(x))] \end{aligned} \tag{2.7}$$

with respect to $s(x)$ and $X(x)$. It is easy to see that optimization with respect to $s(x)$ yields

$$s(x) = c_N . \tag{2.8}$$

As in the previous section, optimization with respect to $X(x)$ yields (2.2a), and the nonnegative-profit condition guarantees (2.2c). Thus the optimal supply of the public good and the optimal level of the congestion tax are obtained.

In the previous section we showed that the marginal benefit of the public good is fully reflected in the increase in land rent. It may seem plausible that, when there is a congestion tax, some of the benefit of an increase in the supply of the public good will show up as an increase in tax revenue, so that the marginal benefit would equal the change in the sum of land rent and the congestion tax. Differentiating the sum, however, yields

$$\begin{aligned}
&\frac{d}{dX}[q(x)R(y-t(x)-s(x),u,X(x))+N(x)s(x)] \\
&= q(x)[-R_I\frac{ds}{dX}+R_X]+N(x)\frac{ds}{dX}+s(x)\frac{dN}{dX} \\
&= q(x)R_X + s(x)\frac{dN}{dX} \\
&= \frac{u_X}{u_z}N(x)+s(x)\frac{dN}{dX}
\end{aligned} \tag{2.9}$$

where the second and the last steps use (2.5) and (2.8) respectively. The change in the sum, therefore, exceeds the marginal benefit, and the difference is the increase in tax revenue caused by an induced change in population, $s(x)(dN/dX)$. The increase in population raises the tax revenue, but at the same time increases the cost of producing the public good. From (2.8), the two increases are equal at the optimum, and the in-crease in tax revenue, being comp letely absorbed by the increased costs, does not constitute net social gain.

(2.9) also shows that, if the congestion tax, $s(x)$, is held constant, the earlier result follows:

$$\begin{aligned}
&\frac{d}{dx}[q(x)R(y-t(x)-s(x),u,X(x))] \\
&= \frac{u_X}{u_z} \qquad\qquad \text{if} \quad s(x) = \text{constant}
\end{aligned}$$

Thus, if, for example, the marginal cost of a population increase, c_N, is constant, the marginal benefit of the public good exactly equals the increase in land rent.

3. A Public Good Local to a City

In this section a local public good is assumed to be jointly consumed by all residents of a city. Consider n identical cities which produce the consumer good under constant returns to scale. A city's production function is wP_c, where P_c is the population of the city and the marginal product of labour, w, is constant. Note that the existence of a public good provides a reason for having a city: an increase in population lowers the *per capita* cost of supplying the same amount of the public good. Cities, therefore, may exist even if production technology has constant returns to scale.

The utility function of a household is

$$u(z(x), h(x), X), \tag{3.1}$$

where X is the consumption of the local public good and is equal for all residents in a city. The cost in terms of the consumer good of the public good is

$$C(X), \tag{3.2}$$

where there is no congestion effect.[7] We do not explicitly introduce a rural sector but the rural rent, R_a, is assumed to be paid by cities. Then the resource constraint is

$$\int_0^{\overline{x}} \{[z(x)+t(x)]N(x)+R_a q(x)\}dx + C(X) = wP_c\,. \tag{3.3}$$

The total population, P, of city residents is assumed to be given. The population constraints are

$$P = nP_c \tag{3.4}$$

and

$$P_c = \int_0^{\overline{x}} N(x)dx\,. \tag{3.5}$$

Our optimization problem is one of maximizing the sum of equal utilities,

$$n\int_0^{\overline{x}} N(x)dx\,, \tag{3.6}$$

under the above constraints (3.3)-(3.5) and the constraint that all households have the same utility level,

$$u(z(x), h(x), X) = u. \tag{3.7}$$

If the number of cities is fixed, we obtain the first order conditions (1.5a), (1.5b), and

$$\int_0^{\overline{x}} \frac{u_X}{u_Z} N(x)dx = C'(x)\,, \tag{3.8a}$$

$$R(\overline{x}) = R_a\,. \tag{3.8b}$$

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7 This formulation implicitly assumes no transportation costs for the local public good. In this sense the public good is like a telephone system, a cable television network or a sewage system but not like a theater or a central park. Transportation costs of a local public good can, however, be easily introduced and do not change our results. If the public good is supplied at the center of the city, we may even interpret $t(x)$ as including transportation costs of the public good.

(3.8a) is the Samuelsonian condition that the sum of marginal rates of substitution over all residents in a city must equal the marginal cost of the public good. (3.8b) is a familiar equality between the urban rent at the edge of a city and the rural rent.

If the number of cities is a policy variable, we must add the following condition:

$$\int_0^{\bar{x}} [R(x) - R_a] q(x) dx = C(X). \tag{3.9}$$

The total differential rent is equal to the total cost of the public good. Therefore, if a city government collects land rent, pays the rural rent, and supplies the local public good, its budget is balanced at the optimal number of cities.

Now, consider the benefit of the public good in a market economy. We first derive a formula which holds for any type of city, and then consider the special cases of a closed city and a small open city in an economy with many cities. In our market cities, city governments are assumed to collect the land rent, and to return the surplus, after the payment of the rural rent and the cost of the public good, to residents as an equal subsidy. Since everybody has the same marginal productivity, the wage rate is also the same, and therefore income is the same for all households. Then the budget constraint for a household is given by

$$y = z(x) + t(x) + R(x)h(x), \tag{3.10}$$

for an appropriate income *y*.

The bid rent function can be derived as in the previous sections:

$$R(x) = R[y - t(x), u, X]. \tag{3.11}$$

The effect on land rent of a change in the supply of the public good is

$$\frac{dR(x)}{dX(x)} = R_I \frac{dy}{dx} + R_u \frac{du}{dx} + R_X$$

$$= \frac{l}{h(x)} \frac{dy}{dx} + \frac{l}{v_I h(x)} \frac{du}{dX} + \frac{l}{h(x)} \frac{u_X}{u_Z}, \tag{3.12}$$

where the second equality is obtained from (2.5), (I.1.15) and (1.10). Multiplying both sides by $q(x)$, integrating from *0* to $\bar{x}$, and rearranging terms, we obtain

$$\int_0^{\bar{x}} \frac{u_X}{u_Z} N(x) dx = \int_0^{\bar{x}} \frac{dR(x)}{dx} q(x) dx + \left[\int_0^{\bar{x}} \frac{l}{v_I} N(x) dx \right] \frac{du}{dx} - P_c \frac{dy}{dx}. \tag{3.13}$$

Thus the marginal benefit of the public good is reflected in the changes of land rent, the utility level and the income level. Notice that this equation holds for any degree of openness of a city.

First, consider the public-ownership case of a single closed city, where the population of the city is fixed. The argument applies as well to an economy with many cities when the number of cities is given. The income of a household satisfies

$$P_c y = P_c w + \int_0^{\bar{x}} [R(x) - R_a] q(x) dx - C(X). \qquad (3.14)$$

Differentiating this equation, and noting that (3.8b) holds in equilibrium, we obtain

$$P_c \frac{dy}{dx} = \int_0^{\bar{x}} \frac{dR(x)}{dX} q(x) dx + [R(\bar{x}) - R_a] q(\bar{x}) \frac{d\bar{x}}{dX} - C'(x)$$

$$= \int_0^{\bar{x}} \frac{dR(x)}{dX} q(x) dx - C'(x). \qquad (3.15)$$

Substituting (3.15) into (3.13) yields

$$\left[\int_0^{\bar{x}} \frac{1}{v_I} N(x) dx \right] \frac{du}{dx} = \int_0^{\bar{x}} \frac{u_X}{u_z} N(x) dx - C'(x), \qquad (3.16)$$

which states that, if the marginal benefit exceeds the marginal cost, the utility level of residents rises as the supply of the public good is increased. At the optimum, where the utility level is maximized, we have

$$\frac{du}{dX} = 0$$

which, coupled with (3.16), yields the Samuelsonian condition (3.8a) for the optimum supply of the public good. Notice that in a closed city the land rent does not necessarily reflect the benefit of the public good.

Next, consider a small, open city. When the number of cities is very large, a city may be considered to be very small. In such a case the utility level of households can be considered as given for a city and (3.13) becomes

$$\int_0^{\bar{x}} \frac{dR(x)}{dX} q(x) dx = \int_0^{\bar{x}} \frac{u_X}{u_z} N(x) dx + P_c \frac{dy}{dX}.$$

Therefore, if the income level is given, an increase in land rent fully reflects the benefits of the public good:

$$\int_0^{\bar{x}} \frac{dR(x)}{dX} q(x) dx = \int_0^{\bar{x}} \frac{u_X}{u_z} N(x) dx. \qquad (3.17)$$

There are at least two such cases. First, if land is owned by absentee landlords, the income of city residents is not affected by the supply of the public good. More important in our context is the case where a central government collects all the fiscal surpluses of city governments and distributes them as an equal subsidy. If a city is small compared to the whole economy, the policy in that city affects the subsidy received by its residents only negligibly, and the income level can be considered as fixed.

The latter case completely parallels the treatment of the extremely local public good case: if a profit-maximizing city developer, owned equally by all households in the economy, supplies the public good, the optimal supply of the public good is achieved. A city developer maximizes the profit

$$\int_0^{\bar{x}} [R(x) - R_a] q(x) dx - C(X)$$

with respect to X among market equilibria. Then at the maximum we have

$$\frac{d}{dx}\left\{\int_0^{\bar{x}}\left[R(x)-R_a\right]q(x)dx-C(X)\right\}$$
$$=\int_0^{\bar{x}}\frac{dR(x)}{dx}q(x)dx+\frac{d\bar{x}}{dX}\left[R(\bar{x})-R_a\right]-C'(x)$$
$$=\int_0^{\bar{x}}\frac{dR(x)}{dx}q(x)dx-C'(X)=0.$$

where derivates are taken across equilibria. Combining this equation with (3.17), we obtain the condition (3.8a) for the optimum supply of the public good. Therefore, a system of land developers does not require that the region be either homogeneous or physically small to achieve an optimum. We do need smallness in the sense that the utility and the income levels of residents are not affected by policies within a city.

If the number of cities is optimal, the profit of a city developer is zero from (3.9). Therefore, the zero profit condition from free entry insures the optimal number of cities. This result parallels those in the cases of increasing returns and Marshallian externality in Chapter II. The main difference is that in the public good case the supply of the public good must be determined, as well as the population of a city, while there is no such variable in previous cases.

In the case of Marshallian externality the market city tended to be too large. This problem does not appear when city formation results from the existence of public goods. Consider the utility level attainable in a city given the allocation in the rest of the world. In the Marshallian externality case the utility level first rose as the population of the city increases, reached a maximum at P_c^*, and then fell as illustrated in Figure 2a.

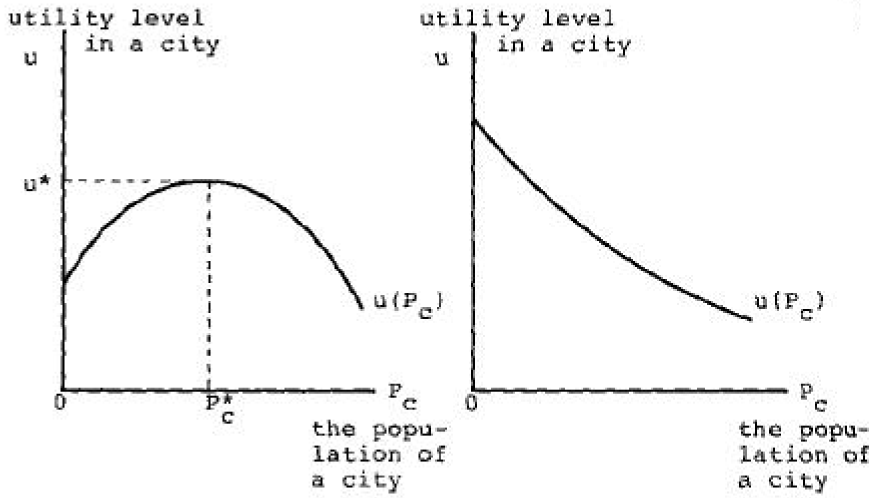

Figure 2a
The Marshallian Externality Case

Figure 2b
The Public Good Case

Figure 2. Comparison with the Marshallian Externality Case

Since the utility level was low when the population was small, it was difficult for a new small city to attract residents. In the public good case, however, the situation is different. For the same supply level of the public good the utility level achievable in a city is higher when the population of the city is smaller as illustrated in Figure 2b. Since the public good is financed by the land rent, residents do not pay any tax for the public good. The residents are therefore better off in a smaller city, since they can enjoy the same amount of the public good with smaller average commuting costs. In such a case a new small city has no trouble attracting residents.

Notes

The analyses in this chapter derive from two separate bodies of literature. The first is concerned with attaining an efficient supply of a local public good. The second with the relationship between land rents and the benefits of public goods.

Samuelson (1954) has shown that it is extremely doubtful that any decentralized market system can determine the optimal level of a pure public good. His main argument is that there is always an incentive to misreveal one's preferences. For local public goods, however, Tiebout (1956) has argued that a decentralized market mechanism can indeed work. Freedom of personal migration among jurisdictions works as voting with one's feet which insures efficiency.

As shown elsewhere (Kanemoto (1976)), this hypothesis is not correct if local governments are passive in supplying local public goods. An argument similar to the discussion of optimum and market city sizes in the Marshallian externality case in Chapter II can be applied to show that, though the optimal; solution is one of market equilibria, there are many other equilibria, and there is no reason to believe that the optimal solution is likely to be attained.

The multiplicity of equilibria occurs since a sudden formation of a new community which is sufficiently large to be viable is usually impossible in a decentralized economy. Therefore, one way to avoid the difficulty is to allow free coalition. As shown by Pauly (1970), however, an efficient allocation is a core only if the total population is divisible by the best community size. Otherwise, a core does not exist. Furthermore, informational requirement to attain a core would be formidable.

Another way of avoiding the difficulty is to introduce an active role of local governments. McGuire (1974) and Berglas (1976) assumed a profit maximizing behaviour of the suppliers of a local public good. They showed that if there are sufficiently many suppliers, an efficient allocation of the public good is attained. For this to be true, however, a firm should be able to determine the number of the members of the club as well as the supply of the public good and the tax (or the membership charge, in their club theory terminology). Though this may be plausible in a club theory, it is usually difficult for a local government to control the population of its jurisdiction. If the population of a community is determined by free migration, the difficulty of forming a sufficiently large new community will remain to be an obstacle to achieving the efficient community size.

If there is a factor whose supply is fixed, notably land, this difficulty disappears. As a local government's policy, Margolis (1968) suggested the principle of fiscal profitability: local governments seek to minimize the burden to the local tax payers. However, he remained doubtful on the optimality of the supply of public goods in a model with the principle of fiscal profitability and voting with one's feet.

Negishi (1972) developed a formal model to analyze this problem and showed that Pareto optimality can be attained under the following three assumptions. First, the marginal rate of substitution between land and local public goods is equal to the reciprocal of the ratio of land inputs to local public goods. Second, local public goods are financed by proportional taxes on land. Third, local governments believe that marginal and average land value productivities of a public good are equal. Unfortunately, these assumptions (especially, the first one) are quite restrictive.

We have shown that Negishi's first and second assumptions are not necessary to establish efficiency of the principle of fiscal profitability coupled with voting with one's feet, if a jurisdiction is very small relative to the whole economy.

The second source of our analysis is the literature on the relationship between land rents and the benefits of public projects. Polinsky and Shavell (1975) and Pines and Weiss (1976) showed that the marginal increase of the land rent in an open and small region correctly reflects the marginal benefit of a public project. Pines and Weiss. added a qualification: if relative prices of goods are affected by the public project (for example, in the case of leisure), this may not be true. We show in Appendix II, however, that, even if the wage rate is affected by the supply of the public good, the marginal benefit is correctly reflected in land rent. We have shown elsewhere (Kanemoto (1978)) that the conclusion holds for models which are still more general than the one used in the appendix, even when leisure is introduced.

The model of an extremely local public good is similar to models in Schuler (1974) and Helpman, Pines and Borukhov (1976).) Their main concern is, unlike ours, the spatial pattern of the supply of the local public good.

The model of a public good local to a city is similar to that of Arnott and Stiglitz (1975) who considered only the optimal allocation. They obtained the result that, in a city with the optimum population, the aggregate land rent equals the total expenditure on public goods. This result was first obtained by Flatters, Henderson, and Mieszkowski (1974) and sometimes called the Henry George Theorem or the Golden Rule. We found that this property follows from the conditions for the , optimal number of cities. It is apparent that the problem of the optimal number of cities is equivalent to the problem of the optimum population of a city in a model with identical cities.

Arnott (1979) discussed market city sizes. His approach, in contrast to ours, was to assume away entrepreneurship of city developers. He therefore repeated the argument which Henderson (1974) gave in the case of Marshallian externality and concluded that the market city size tends to be greater than the optimum.

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