CHAPTER V

TRAFFIC CONGESTION AND LAND USE FOR TRANSPORTATION: THE SECOND BEST CITY[1]

In the previous chapter we introduced traffic congestion and analyzed the optimum and market allocations. With transportation congestion, an additional traveler imposes external costs on other travelers by slowing them down. The optimal solution requires congestion tolls to "internalize" this externality. It is, however, difficult to charge congestion tolls because of very high administrative costs. In fact, there are very few roads where congestion tolls are levied and there is no city where congestion tolls are adopted in the whole city. It is, therefore, very important to consider what can be done given the constraint that congestion tolls are not allowed.

In the market city of the preceding chapter, we assumed that roads are built according to a naive benefit-cost criterion: the direct saving in transportation costs from widening the road is equated to the market land rent. This benefit-cost criterion leads to a misallocation of land between transportation and residential uses since, given the absence of congestion tolls, the market rent does not correctly reflect the true social scarcity of land.

In this chapter we consider the second best problem, which is to optimize the allocation of land between roads and residence when congestion tolls are not levied. The benefit-cost criterion that must be adopted to achieve the second best allocation is more complicated than the one in the optimum city or the market city. The cost side must be the *shadow rent*, or the *social rent*, which is no longer equal to the market rent. The benefit side also differs from the marginal direct saving in transportation cost (unless compensated demand for land is completely price or rent inelastic). The reason is as follows. A reduction of transportation costs from widening the road induces a change in the market rent. If demand for land is responsive to a price change, this has a side effect of changing the consumption decisions of households. As shown in the previous chapter, the social value of the change is zero due to the envelope property if the market rent equals the social rent. In the second best city, however, the market rent is not equal to the social rent, and a change in the consumption decision results in a net social gain or loss. The loss or gain is the difference between the social benefit and the marginal reduction in transportation costs.

Since the naive benefit-cost criterion usually adopted by policy makers leads to a suboptimal allocation of land, it is of interest to know the *direction* of the misallocation, that is, whether there is overinvestment or underinvestment in roads. The direction of

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[1] This chapter is based on my 1977 paper in the Journal of Urban Economics. I would like to thank Academic Press, Inc. for permitting me to include an extended version of the paper in this book.

misallocation may be determined by comparing the market city with the second best city, but the second best city is, unfortunately, so complicated that we have not been able to carry out the comparison directly. We therefore examine the direction of a change that the naive benefit-cost criterion suggests at the second best optimum. More specifically, we compare the marginal saving in transportation costs and the market rent when roads are built in the second best way.

This comparison yields unambiguous results only if the benefit-cost criterion is adopted in a small region while the allocation in the rest of the city is held constant. The criterion leads to overinvestment in roads if the marginal saving in transportation costs is greater than the market rent at the second best optimum. If, however, the criterion is adopted in the entire city, interrelationships among different locations introduce complicated reactions, and we cannot obtain a definite answer.

In the second best city, the market rent at the edge of the city does not equal the rural rent although the shadow rent does. This result is in sharp contrast to those obtained in the optimum and market cities. The city must be expanded out to the radius where the contribution of an additional unit of land equals the rural rent. This requires the shadow rent to be equal to the rural rent. Since the market rent equals the shadow rent in the optimum city, the market rent also equals the rural rent at the edge of the city. In the second best city, however, the market rent is no longer equal to the shadow rent and hence is not equal to the rural rent at the edge.

Imposing another constraint that the market rent equals the rural rent at the edge of the city does not essentially change the situation. It is always possible, for instance, to make the width of the road zero and transportation costs per mile infinite at the edge of the city. This can cause a sudden drop in the market rent profile at the city's edge so that the market rent equals the rural rent after the drop and the constraint can be satisfied without changing the allocation inside the city. The only way to make the constraint significant is to restrict the shape of the road width functions, for example, to the class of linear functions as in Solow (1973).

The case where compensated demand for land is completely price inelastic is peculiar in the following two respects. First, the social marginal benefit of the road equals the direct marginal saving in transportation costs, since a change in rent caused by widening the road does not induce any change in consumption decision. Second, the absolute level of the market rent is indeterminate as long as difference in rents at different locations is such that the utility levels are equal. The second property mislead Solow and Vickrey (1971) and Kanemoto (1975) to conclude that the market rent is lower than the shadow rent everywhere in the city. In this case, there is no need for a jump in the market rent to make the market rent equal to the rural rent at the edge of the city, since the level of the market rent is indeterminate. This, coupled with the result that the slope of the shadow rent is steeper than that of the market rent, implies that the market rent is lower than the shadow rent everywhere in the city. This result, however, is misleading since it does not carry over to the case where the elasticity is not zero even when the elasticity is extremely small.

This chapter is organized as follows. The model is set up in section 1. Section 2 is the largest section in this chapter and devoted to the case of a closed city. The section is divided into three subsections: in subsection 2.1 the first order conditions for the second best optimum are derived and interpreted, in subsection 2.2 the benefit (the direct marginal saving in transportation costs) and cost (the market rent) of the naive benefit-cost criterion based on market prices are compared at the second best optimum, and in subsection 2.3 the case of completely inelastic demand for land is considered. An open city is analyzed in section 3, and an economy consisting of many cities in section 4.

1. The Model

In this chapter we make the same technological assumptions as in Chapter IV. The only difference lies in the nature of the optimization problem: in this chapter congestion tolls are not allowed but the width of the road is optimized, whereas in the optimum city both congestion tolls and the width of the road could be chosen, and in the market city congestion tolls were not allowed and the road was built according to the erroneous benefit-cost criterion based on market prices.

Since congestion tolls are not allowed, households pay the private (or average) transportation cost, $t(x)$, defined by (IV.1.6) and (IV.1.7):

$$t'(x) = g(T(x), L_T(x)), \tag{1.1}$$

$$t(0) = 0. \tag{1.2}$$

If we denote the income of a household by y and the rent at x by $R(x)$, a household at x maximizes the utility function, $u(z(x), h(x))$, under the budget constraint

$$y = z(x) + R(x)h(x) + t(x). \tag{1.3}$$

Because of spatial arbitrage, the rent function, $R(x)$, must be such that the utility levels are equal everywhere in the city. As in section I.1.1, all this information can be summarized in the bid rent function,

$$R(x) = R(y - t(x), u), \tag{1.4}$$

which satisfies (I.1.14) and (I.1.15):

$$R_I(y - t(x), u) = 1/h(x), \tag{1.5}$$

$$R_U(y - t(x), u) = -1/v_I h(x), \tag{1.6}$$

where u is the equal utility level. Consumptions of the consumer good and housing are given by the compensated demand functions,

$$z(x) = z(R(x), u), \tag{1.7}$$

$$h(x) = h(R(x), u), \tag{1.8}$$

which satisfy (I.1.19) and (I.1.20):

$$z_R(R(x), u) \geq 0, \tag{1.9}$$

$$h_R(R(x), u) \leq 0. \tag{1.10}$$

The volume of traffic at x, $T(x)$, satisfies (IV.1.9) and (IV.1.10):

$$T'(x) = -L_H(x)/h[R(y-t(x),u),u], \tag{1.11}$$

$$T(\bar{x}) = 0. \tag{1.12}$$

The widths of the residential area and the road must satisfy the land constraint (IV.1.1):

$$L_H(x) + L_T(x) \leq q(x)\,. \tag{1.13}$$

2. A Closed City

In a closed city the population of the city is fixed, which yields the boundary condition (IV.2.1) for $T(x)$ at $x = 0$:

$$T(0) = P. \tag{2.1}$$

Using (1.4), (1.7), (1.8) and a different representation of transportation costs (tN instead of Tg), we can rewrite the resource constraint (IV.2.2) as

$$\int_0^{\bar{x}} \{ \frac{z[R(y-t(x),u),u] + t(x)}{h[R(y-t(x),u),u]} L_H(x) + R_a q(x) \} dx \leq Pw. \tag{2.2}$$

2.1. Derivation and Interpretation of First Order Conditions

In the second best problem the distortion of relative prices caused by the absence of congestion tolls is taken as given. The bid rent function (1.4) and demand functions, (1.7) and (1.8), of the consumer good and land capture the response of households to this distortion. Thus the second best problem maximizes the sum of utilities, (IV.2.3),

$$\int_0^{\bar{x}} \frac{uL_H(x)}{h[R(y-t(x),u),u]} dx, \tag{2.3}$$

under the constraints (1.1), (1.2), (1.11), (1.12), (1.13), (2.1), and (2.2). There are two state variables in this problem: $t(x)$ and $T(x)$. The control variables are $L_H(x)$ and $L_T(x)$. The control parameters are y, u, $\bar{x}$, and $t(\bar{x})$.

We assume that the market rent at the edge of the city, $R(\bar{x})$, is not restricted to equal the rural rent. In this case, there is no constraint on $t(\bar{x})$. The constraint on the market rent at the edge of the city does not cause any essential difference in the optimum allocation if we assume that transportation costs per mile become infinite, as the width of the road tends to zero. Under this assumption it is possible to have the same allocation inside the city and at the same time to satisfy the constraint by causing a jump in the market rent. Since the difference in allocation occurs only in an infinitesimal interval, this is possible without violating the resource constraint. Thus the constraint on the market rent is superfluous.

The Hamiltonian for the second best problem is

$$\Phi = [u - l(x)]\frac{L_H(x)}{h[R(y-t(x),u)u]} + h(x)g(T(x), L_T(x))$$
$$- d\{\frac{z[R(y-t(x),u),u] + t(x)}{h[R(y-t(x),u),u]}L_H(x) + R_d q(x)\}, \qquad (2.4)$$

where $l(x)$, $h(x)$, and d are respectively adjoint variables associated with (1.11), (1.1), and (2.2). Forming the Lagrangian,

$$\Psi = \Phi + m(x)[q(x) - L_H(x) - L_T(x)], \qquad (2.5)$$

where $m(x)$ is the Lagrange multiplier for the constraint (1.13), we obtain the following necessary conditions for the optimum:

$$\frac{\partial \Phi}{\partial T} = -l'(x) = hg_T(T, L_T), \qquad (2.6)$$

$$\frac{\partial \Phi}{\partial t} = -h'(x)$$
$$= -dN + [u - l - d(z+t)]\frac{N}{h}h_R R_I + dNz_R R_I \qquad (2.7)$$

$$\frac{\partial \Psi}{\partial L_H} = [u - l - d(z+t)]\frac{1}{h} - m = 0, \qquad (2.8)$$

$$\frac{\partial \Psi}{\partial L_T} = hg_L(T, L_T) - m = 0, \qquad (2.9)$$

where m and d satisfy

$$m(x)[q(x) - L_H(x) - L_T(x)] = 0, \qquad m(x) \ge 0 \qquad (2.10)$$

$$d\{Pw - \int_0^{\bar{x}}[zL_H/h + Tg + R_a q]dx\} = 0, \qquad d \ge 0. \qquad (2.11)$$

The transversality conditions for $\bar{x}$, $t(\bar{x})$, u, and y are

$$\Phi(\bar{x}) = [u - l(\bar{x}) - d(z(\bar{x}) + t(\bar{x}))]N(\bar{x}) - dR_a q(\bar{x})$$
$$+ h(\bar{x})g(T(\bar{x}), L_T(\bar{x})) = 0, \qquad (2.12)$$

$$h(\bar{x}) = 0 \qquad (2.13)$$

$$\int_0^{\bar{x}}\{N - [u - l - d(z+t)]\frac{N}{h}[h_R R_u + h_u] - dN[z_R R_u + z_u]\}dx = 0, \qquad (2.14)$$

$$\int_0^{\bar{x}}\{[u - l - d(z+t)]\frac{N}{h}h_R R_I + dNz_R R_I\}dx = 0. \qquad (2.15)$$

For convenience, we divide the shadow prices, l, h, m, and the utility, u, by d, and substitute the original notations for the variables obtained. This operation converts the shadow prices from utility terms into pecuniary terms. Substituting (2.8) into (2.7), and noting that the rent function and the compensated demand functions satisfy both (1.5) and

$$Rh_R + z_R = 0\,,$$ [2] (2.16)

we can rewrite (2.7) as

$$-h'(x) = -N - \frac{m-R}{R}eN\,, \qquad (2.17)$$

where e is the price (rent) elasticity of compensated demand for land defined by

$$e = -\frac{Rh_R}{h} \geq 0\,. \qquad (2.18)$$

The inequality is obtained because the substitution effect, h_R, is always nonpositive as in (1.10). Notice that e is a function of R and u and hence in general varies over space.

From (2.7), $-h'(x)$ can be interpreted as the social benefit of a unit increase of the commuting costs, $t(x)$, of residents living at x. When $t(x)$ increases by one unit, the total commuting costs are paid by $N(x)$ households who are living at x. This is represented by the first term on the RHS of (2.17). In addition to this direct effect, the increase of $t(x)$ has a side effect on the consumption decisions of households. The market rent, $R(x)$, must fall to compensate the increase of the commuting costs, $t(x)$, which induces a change in consumptions of housing and the consumer good. The second term on the RHS of (2.17) captures this indirect effect.

By the envelope property the second term vanishes when the social rent is equal to the market rent. The envelope property, (2.16), insures that, in the neighborhood of the equilibrium (or optimal) point, the changes in consumptions of the two goods evaluated at market prices counteract each other. In the first best world, therefore, where market prices reflect social values, the social cost of a unit increase of $t(x)$ is $N(x)$.

There is another case where the second term vanishes. When housing demand is completely price inelastic, $e = 0$, the change of the rent does not affect the consumption decision. Therefore, there is no side effect even when the social rent is different from the market rent. This is also a first best situation because the decisions of households are not affected by the existence of congestion and the first best solution can be attained without congestion tolls.

(2.17) shows that the adjustment of consumption has a socially desirable effect if R is greater than m, which makes sense intuitively. An increase in commuting costs, $t(x)$, lowers the market rent, $R(x)$. When the market rent is higher than the social rent, a fall in the market rent brings it closer to the social rent, and the adjustment of

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[2] This can be shown as follows. By the definition of compensated demand functions, $h(R,u)$ and $z(R,u)$ must satisfy

$$u = u\left[h(R,u), z(R,u)\right],$$

for any R. Differentiating both sides with respect to R, we obtain

$$u_h h_R + u_z z_R = 0\,.$$

Since $u_h / u_z = R$, this implies

$$Rh_R + z_R = 0\,.$$

consumption works in the socially desirable direction.

Integrating (2.17) from x to $\bar{x}$ and using the transversality condition (2.13), we obtain

$$h(x) = -T - \int_x^{\bar{x}} \frac{m-R}{R} eNdx' . \tag{2.19}$$

Thus, $-h(x)$ is the social cost of increasing commuting costs of all households living between x and $\bar{x}$ by one unit.

Using this interpretation of $h(x)$, we can interpret $l'(x)$ in (2.6) as the social congestion cost due to a unit increase in traffic. A unit increase in traffic between x and $x+dx$ causes more congestion there and raises transportation costs to pass through the ring by $gT(T, L_T)dx$. Since all households living beyond the ring must pass through the ring, the social cost of this increase in transportation costs is approximately $-h(x)g_T dx = l'(x)dx$.

From (2.8) and the budget constraint (1.3), we have

$$m(x) = R(x) + \frac{u - y - l(x)}{h(x)} , \tag{2.20}$$

where $m(x)$ is the shadow rent of land at x and the right hand side is the marginal value of land in residential use. The shadow rent differs from the market rent, and hence from the marginal rate of substitution between housing and the consumer good. The difference is caused by the second term on the right side, which reflects the congestion costs.

From (2.9) the shadow rent $m(x)$ also satisfies

$$m(x) = h(x) g_L(T(x), L_T(x)) . \tag{2.21}$$

The right side can be interpreted as the social marginal value of land in transportation use. A marginal increase of land allocated to roads lowers transportation costs at the radius. The social value of this decrease is given by the right side of (2.21).

From (2.6) and (2.21), we obtain

$$l'(x)T - mL_T = -h[Tg_T + L_T g_L] ,$$

where, as shown in subsection 2.1 of Chapter IV, the square bracket on the right side is negative if transportation technology exhibits increasing returns to scale and positive in the case of decreasing returns. Since g_L is negative and $m(x)$ is nonnegative, (2.21) implies that $h(x)$ is nonpositive. Thus the following relationships hold between the total social congestion costs and the total shadow rent of roads at any radius:

$$l'(x)T(x) \begin{matrix} < \\ = \\ > \end{matrix} mL_T \quad \begin{matrix} \text{in the increasing returns case} \\ \text{in the constant returns case} \\ \text{in the decreasing returns case.} \end{matrix} \tag{2.22}$$

This result is more general than the condition obtained for the first best solution, where the relationship was expressed in terms of the actual congestion tolls and the road rent.

Using (2.19), we can rewrite (2.21):

$$m(x) = B(x) - g_L(T, L_T) \int_x^{\bar{x}} \frac{m - R}{R} eN dx' , \tag{2.23}$$

where

$$B(x) \equiv -T g_L(T, L_T) \tag{2.24}$$

is the marginal direct saving in transportation costs from widening the road as defined by (IV.2.20), and is sometimes called the *market benefit*. The second term on the right of (2.23) represents the social cost of the adjustment in the consumption of land for housing, which is characteristic of the second best world.

The naive benefit-cost criterion based on market prices cannot achieve the second best allocation of land. Although the social marginal values of land in residential and transportation uses are equal at the second best optimum, the market rent of the residential land is not in general equal to the market benefit of the road, since the market values differ from the social values as shown in (2.20) and (2.23).

It is easy to see that the transversality conditions, (2.12) and (2.13), imply that the shadow rent equals the rural rent at the edge of the city:[3]

$$m(\bar{x}) = R_a . \tag{2.25}$$

The transversality condition, (2.15), can be written more simply:

$$\int_0^{\bar{x}} \frac{m - R}{R} eN dx = 0 . \tag{2.26}$$

Though this equation is very important in deriving qualitative results (it is used in both Theorem 1 and Theorem 2 below), it is difficult to provide an interesting interpretation.

(2.14) can be simplified by using uncompensated demand functions for land and for the consumer good, $\hat{h}(I, R)$ and $\hat{z}(I, R)$, defined in (I.1.5) and (I.1.6) respectively. Compensated and uncompensated demand functions satisfy the following relationships derived in (3.14) and (3.16) of Appendix III.

$$h_u v_I = \hat{h}_I$$

$$z_u v_I = \hat{z}_I$$

$$\hat{h}_R = h_R - h\hat{h}_I .$$

From these equations and (I.1.9), (1.6), and (2.16), (2.14) can be written

$$\int_0^{\bar{x}} \left[\frac{1}{d} - \frac{1}{v_I} \right] N dx = -\int_0^{\bar{x}} [m - R] \frac{\hat{h}_R}{h} \frac{N}{v_I} dx . \tag{2.27}$$

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[3] In deriving this condition, we assumed that g is finite at $\bar{x}$. It seems very unlikely that g becomes infinite at $\bar{x}$ because traffic is very light and available land is very large there.

This equation describes the relationship between the social value of the numeraire good (d) in utility terms and the marginal utility of income (v_I). When there is no congestion, the right side vanishes and we obtain (I.2.23d) which says that the averages of reciprocals of these two are equal. When the shadow rent is not equal to the market rent, the reciprocal averages differ by the term on the right.

From (2.20) and (2.23), the benefit-cost criterion that must be used to achieve the second best allocation differs from the naive one adopted in the optimum and market cities. Unfortunately, it is not easy to calculate the correct benefit and cost. We can express the difference, $r(x)$, between the shadow rent, which represents the correct social cost, and the market rent by

$$r(x) \equiv m(x) - R(x) = [u - y - l(x)]/h(x). \tag{2.28}$$

The difficulty is that the values of u and $l(x)$ are not directly observable. The policy maker can, however, observe $h(x), N(x), T(x), L_T(x)$, and $R(x)$ without too much difficulty, and can estimate, with some more difficulty, the compensated price elasticity, $e(x)$. We therefore express $r(x)$ in terms of these variables. From (2.6) and (2.19), $r(x)$ satisfies

$$\begin{aligned} &r'(x)h(x) + r(x)h'(x) \\ &= \left[T(x) + \int_x^{\bar{x}} \frac{r(x')}{R(x')} e(x')N(x')dx' \right] g_T(T(x), L_T(x)), \end{aligned} \tag{2.29}$$

and from (2.26),

$$\int_0^{\bar{x}} \frac{r(x)}{R(x)} e(x)N(x)dx = 0. \tag{2.30}$$

The difference between the shadow rent and the market rent can be calculated by solving the differential equation (2.29) with the boundary condition (2.30). Then the social marginal cost of widening the road is simply the sum of the difference, $r(x)$, and the market rent, $R(x)$. Although it is not extremely difficult to solve the differential equation numerically in simple models like ours, the calculation is likely to be formidable in a more realistic model.

Once the difference between the shadow rent and the market rent is obtained, the social benefit can be easily calculated from (2.23):

$$B(x) - g_L(T, L_T) \int_x^{\bar{x}} \frac{r(x')}{R(x')} e(x')N(x')dx'.$$

2.2. Comparison of the Market Benefit and the Market Rent

Having simplified and interpreted first order conditions, we can now proceed to examine the consequence of the benefit-cost analysis based on market prices. Our ultimate goal is to compare the market benefit, $B(x)$, and the market rent, $R(x)$, at the second best optimum. It is convenient to compare the social rent, $m(x)$, with each of these first.

In this subsection we consider the case where compensated demand for land is not completely price inelastic: $e > 0$.

The social rent is equal to the market rent in the optimum city with optimal congestion tolls. If, however, congestion t tolls are not levied, the market rent diverges from the social rent. Since transportation costs are lower than they should be,

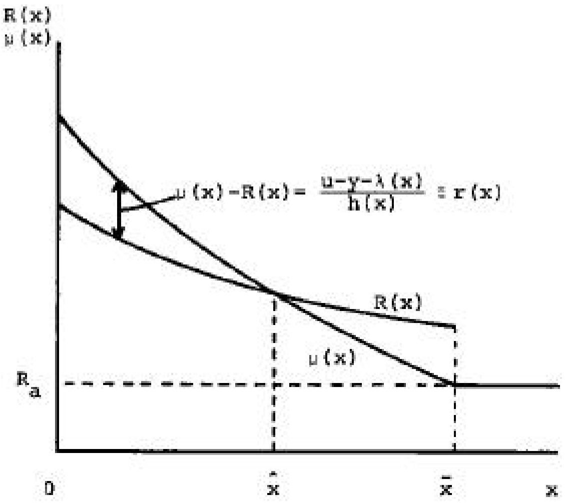

Figure 1. The Relationship between the Market Rent and the Social Rent

Households tend to locate too far from the CBD. People seeking land farther from the center bid up the rent at larger radii, and the market rent tends to be *flatter* than the social rent. The following Theorem shows that the market rent crosses the social rent at some intermediate radius, and that the social rent must be higher than the market rent inside the radius and lower outside the radius. This is illustrated in Figure 1.

Theorem 1: If $e > 0$ *for any radius, then there exists an* $\hat{x}$ *strictly between 0 and* $\bar{x}$ $(0 < \hat{x} < \bar{x})$ *such that* $m(\hat{x}) = R(\hat{x})$*, and*

$$m(x) > R(x) \qquad \textit{for} \quad 0 \le x < \hat{x},$$

$$m(x) < R(x) \qquad \textit{for} \quad \hat{x} < x \le \bar{x}.$$

Proof:

From (2.26) and $e > 0$, it is impossible to have $m(x) > R(x)$ for all x or $m(x) < R(x)$ for all x. Since both $m(x)$ and $R(x)$ are continuous, they must cross somewhere: there exists an $\hat{x}$, $0 \le \hat{x} \le \bar{x}$, where $m(\hat{x}) = R(\hat{x})$. From (2.20), at this point $l(x)$ satisfies

$$l(\hat{x}) = u - y .$$

From (2.6), (2.9), (2.10), (IV.1.3), and (IV.1.4), we obtain

$$l'(x) = -mg_T / g_L > 0 .$$

This inequality is strict at $\hat{x}$ since $m(\hat{x}) = R(\hat{x}) > R_a > 0$. Hence we obtain the following inequalities:

$$l(x) < u - y \qquad x < \hat{x}$$

$$l(x) > u - y \qquad \hat{x} < x.$$

From (2.20), these inequalities imply

$$m(x) > R(x) \qquad x < \hat{x}$$

$$m(x) < R(x) \qquad \hat{x} < x,$$

which in turn implies that $\hat{x}$ must be strictly between *0* and $\hat{x}$ to satisfy (2.26).

Q.E.D.

We next compare the market benefit and the social rent. The next Theorem shows that they are equal at $x = 0$ and that the market benefit is greater than the social rent in the rest of the city. Thus the market benefit overestimates the true social benefit. This is illustrated in Figure 2.

The result can be understood intuitively as follows. Recall that the difference between the market benefit and the social rent is the social value of the adjustment of consumptions in response to a decrease in transportation costs. First, consider the social value of the adjustment caused by a transportation improvement at $x = 0$. The improvement reduces commuting costs for all households by the same amount, which is equivalent to

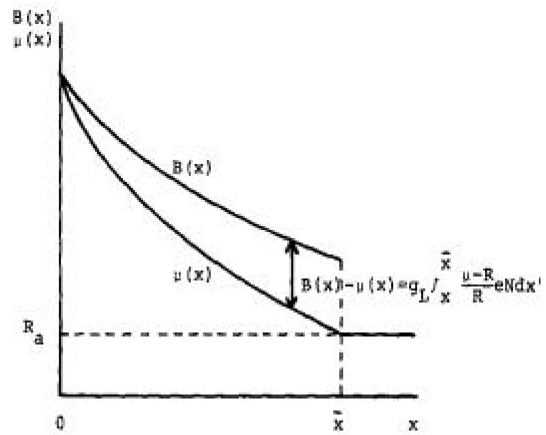

Figure 2. The Relationship between the Market Benefit and the Social Rent

an increase in the income, *y*, of every household in the city. Since *y* is optimally chosen, the change in the utility level caused by an infinitesimal increase in y is zero. The social value of the consumption adjustment is, therefore, zero for an improvement at $x = 0$.

Next, consider an improvement at any radius x beyond $\hat{x}$ in Theorem 1. This decreases commuting costs of households living farther than *x* and raises the market rent there. Since the social rent is lower than the market rent beyond $\hat{x}$, this works in a socially undesirable direction and causes a social loss. Thus the social benefit (and hence the social rent) is less than the market benefit at any radius beyond $\hat{x}$.

An improvement inside $\hat{x}$ benefits both households living outside $\hat{x}$ and inside $\hat{x}$. The consumption adjustments of households outside $\hat{x}$ cause social losses for the same reason as above, but those of households inside $\hat{x}$ are socially beneficial since the social rent is higher than the market rent there. The next Theorem shows, however, that the former is always greater than the latter except for an improvement at $x = 0$ in which case the two are equal.

Theorem 2: If $e > 0$ *for any x, then we obtain*

$$m(0) = B(0)$$

and

$$m(x) < B(x)\,, \qquad \textit{for} \qquad 0 < x \leq \bar{x}\,.$$

Proof:

We first show that for any x strictly between 0 and $\bar{x}$,

$$\int_x^{\bar{x}} \frac{m - R}{R} eNdx' < 0\,.$$

For x greater than or equal to $\hat{x}$, this can be immediately obtained since $m(x) < R(x)$ from Theorem 1. For x less than $\hat{x}$ this is obtained from

$$\int_x^{\bar{x}} \frac{m - R}{R} eNdx' = -\int_0^x \frac{m - R}{R} eNdx' < 0.$$

Hence (2.23) yields

$$m(x) < B(x) \qquad\qquad 0 < x < \bar{x}\,.$$

At $x = 0$, the following equality is obtained:

$$m(0) = B(0) - g_L(T(0), L_T(0)) \int_0^{\bar{x}} \frac{m - R}{R} eNdx$$

$$= B(0)\,,$$

where the second equality is obtained from (2.26), since g_L can be seen to be finite at $x = 0$.

At $x = \bar{x}$, however g_L becomes infinite and we must use *L'Hôpital's Rule* to obtain

$$\begin{aligned}
&\lim_{x\to\bar{x}} - g_L \int_x^{\bar{x}} \frac{m-R}{R} eNdx' \\
&= \lim_{x\to\bar{x}} - \frac{m(x)}{h(x)} \int_x^{\bar{x}} \frac{m-R}{R} eNdx' \\
&= R_a \lim_{x\to\bar{x}} \frac{1}{h'(x)} \frac{m-R}{R} eN \\
&= \frac{R_a\left[R_a - R(\bar{x})\right]e}{R(\bar{x}) + \left[R_a - R(\bar{x})\right]e} \\
&< 0 \ ,
\end{aligned}$$

where the first equality is obtained from (2.9), the second equality by *L'Hôpital's Rule*, the third equality from (2.17), and the inequality from $R(\bar{x}) > m(\bar{x}) = R_a$ and the elementary result that the limit must be nonpositive when it is approached through nonpositive values. From (2.23) this implies

$$m(\bar{x}) < B(\bar{x})$$

Q.E.D.

Combining Theorems 1 and 2, we can immediately see that the market benefit is greater than the market rent near the center. However, it is not clear whether or not this remains to be true when we move farther from the center. The next proposition throws a light on this question.

Proposition 1: If the compensated demand for land is not completely price inelastic $(e > 0)$, *then the market benefit is always greater than the market rent near the CBD. Near the edge of the city, however, the market benefit is smaller than the market rent if the price elasticity is less than one, and is greater than the market rent if the elasticity is greater than one.*

Proof:

The first half is immediately obtained from Theorem 1 and 2.

From the proof of Theorem 2, we obtain

$$\begin{aligned}
R(\bar{x}) - B(\bar{x}) &= R(\bar{x}) - R_a + R_a - B(\bar{x}) \\
&= \frac{\left[R(\bar{x}) - R_a\right]R(\bar{x})(l-e)}{R(\bar{x}) + \left[R_a - R(\bar{x})\right]e} .
\end{aligned}$$

Noting that the denominator and the square bracket of the numerator are both positive, we get

$$R(\bar{x}) \mathrel{\substack{<\\=\\>}} B(\bar{x}) \quad \text{where} \quad e \mathrel{\substack{<\\=\\>}} 1 .$$

Q.E.D.

Figure 3 illustrates the relationship between the market benefit and the market rent in the case of price inelastic demand for land: the market benefit is greater than the market rent near the center of the city, but drops below it near the edge. As a result, the naive benefit-cost criterion has a tendency to overinvest in roads near the center and to underinvest near the edge. When demand for land is price elastic as in Figure 4, the benefit-cost criterion tends to overinvest in roads both near the center and near the edge of the city[4].

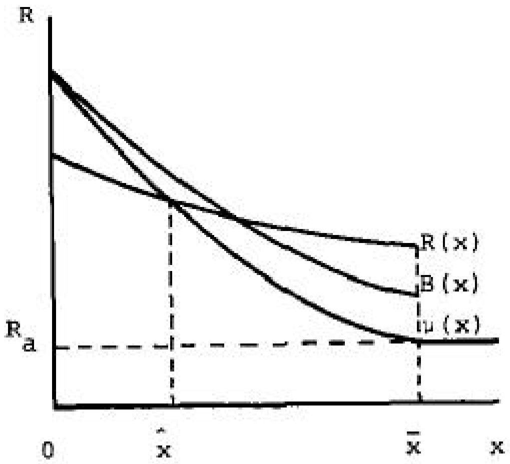

Figure 3
Price Inelastic Case: e < 1

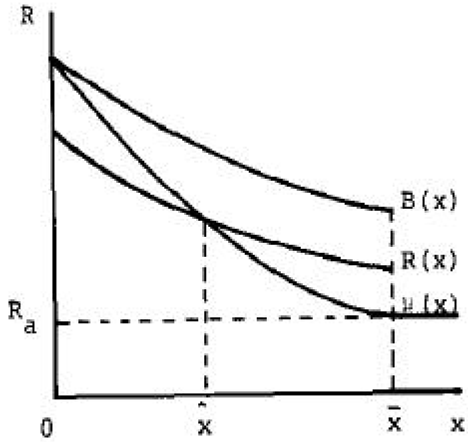

Figure 4
Price Elastic Case: e > 1

Since the Cobb-Douglas type utility function (IV.2.27) has the elasticity $1-a$, which is always less than *1*, there is a tendency in that case to overinvest in roads near the center and to underinvest near the edge.

The conclusion depends on the elasticity of demand for land since difference between the market benefit and the social rent reflects the side effect due to the change of housing consumption, and the change of housing consumption is greater when the elasticity is bigger.

Notice that since these results are valid only in the neighbourhood of the second best solution, we do not have a definite answer as to whether the second best solution has a wider road than the market solution.

When the naive benefit-cost analysis based on market prices is adopted only in a small ring at x, and roads are built in other parts of the city to achieve the second best allocation, the above comparison between two equilibria is valid. If, for example, the market benefit is greater than the market rent in the ring between x and $x+dx$, the naive criterion calls for the road to be widened until the marginal market benefit of further widening falls to the market rent. When the ring is very narrow the market rent is not significantly affected by a change in road width there, and the preceding conclusions hold.

If, however, the naive benefit-cost criterion is adopted in the entire city, this

4 Note that the case where $B(x)$ is lower than $R(x)$ somewhere in the middle of the city is not excluded.

argument cannot be applied because the market rent curve changes. Widening of the road in the rest of the city might cause such a rise in market rent at some locations that, even though the market rent at the second-best allocation was below the market benefit, the road might become narrower as a result of changes elsewhere.

Furthermore, since the market rent is higher than the rural rent at the edge of the city, the city tends to expand. This causes another tendency toward overinvestment in roads. The reader may think that this effect would not appear if the second best problem were solved with the additional constraint that the market rent equal the rural rent at the edge of the city. In our model, however, under the reasonable assumption that transportation costs per mile, $g(T, L_T)$, are infinite when the width of the road is zero, the constraint is superfluous and the effect does not disappear.

The constraint on the market rent at the boundary,

$$R(y - t(\bar{x}), u) = R_a, \tag{2.28}$$

would restrict y, $t(\bar{x})$, and u to a hypersurface. The optimum allocation for the problem with this additional constraint is essentially the same as that for the problem without the constraint: the allocation is exactly the same within the boundary $\bar{x}$, and $g(T, L_T)$ is made infinite at $\bar{x}$ causing a jump in $t(x)$ of an appropriate size to satisfy the constraint (2.28). Since the jump which occurs in an infinitesimally small interval does not involve a finite social cost, the same maximum without the constraint is attained.[5]

Now, we briefly consider the possibility that $t(x)$ has jumps even without the constraint (2.28). In such a case the usual maximum principle like the Theorem of Hestenes in Appendix IV cannot be applied since it assumes that state variables are continuous. Kanemoto (1977b) analyzed the case by considering the problem with an upper bound on $g(T, L_T)$ and letting the upper bound tend to infinity.

The following argument shows that a jump in $t(x)$ is indeed possible. Equation (2.21) suggests that $h(x)$ must be nonpositive, since $m(x)$ is nonnegative. There is no guarantee, however, that $h(x)$ is nonpositive since $h(x)$ must also satisfy (2.19). If compensated demand for housing is sufficiently price elastic, the indirect benefit from increasing transportation costs (the second term on the right side of (2.19)) may overwhelm the direct cost (-*T*), in which case $h(x)$ becomes positive. Then the necessary conditions for the optimum involve contradiction, which suggests that the maximum does not exist within the range of functions assumed by the maximum principle.

In order to show that such a case can occur, we rewrite (2.19) as

$$h(x) = -\int_x^{\bar{x}} \left[1 + \frac{m - R}{R} e\right] N dx'.$$

---

[5] It can be shown that, if g is infinite when L_T is zero, then a jump in $t(x)$ may occurs at $\bar{x}$. See Kanemoto (1977b). Although the proof there has a minor error, the conclusion can be easily seen to be correct.

This equation shows that, if $e(\bar{x}) > R(\bar{x})/[R(\bar{x}) - R_a]$, $h(x)$ is positive near $\bar{x}$. In particular, if $R_a = 0$ and $e(\bar{x}) > 1$, then $h(x)$ is positive. There certainly exists a well-behaved utility function whose compensated demand function is price elastic.

In Kanemoto (1977b) it was shown that, if $g(T, L_T)$ tends to infinity as traffic density, T/L_T, approaches infinity, a jump in $t(x)$ occurs at a point where $h(x)$ is positive. Theorem 1 remains valid even when a jump occurs. Theorem 2 and Proposition 1 are also valid if R_a and $R(\bar{x})$ are replaced by the left side limits,

$$m^-(\bar{x}) = \lim_{x \uparrow \bar{x}} m(x),$$

$$R^-(\bar{x}) = \lim_{x \uparrow \bar{x}} R(x).$$

If g remains finite even when T/L_T approaches infinity, L_T becomes zero for a finite length. It can be easily seen that if the upper bound for g is sufficiently large, the same results are obtained.

2.3. Completely Price Inelastic Demand for Land

Next, consider the case where the compensated demand for land is completely price inelastic: $e = 0$ for any *u* and *R*. This case is obtained, for example, if the utility function is a Leontief type, so that land and the consumer good are always consumed in fixed proportions.

As we mentioned in subsection 2.1, the side effect due to the adjustment of consumption decisions vanishes in this case,

$$h(x) = -T(x),$$

and the market benefit coincides with the social rent,

$$m(x) = B(x), \qquad 0 \le x \le \bar{x}.$$

Since (2.26) is satisfied at all levels of $R(x)$, the level of $R(x)$ is indeterminate. This can be understood as follows. Suppose that the optimum is obtained by the rent function, $R^*(x)$. Consider the effect of raising the rent function by an arbitrary amount c everywhere in the city. Since the utility level cannot be higher than the optimal level, if we can show that the optimal utility level is attained even when the market rent is $R^*(x) + c$, we can conclude that the market rent is indeterminate at the optimum.

When the utility level is given, the assumption of completely inelastic demand implies that lot sizes are constant regardless of the market rent. This has two implications: the lot size is the same everywhere in the city, and it does not change when the rent profile rises to $R^*(x) + c$. In our model differential rent is returned to residents as an equal subsidy, so the income of households rises by ch^*, where h^* is the optimal lot size. Households, therefore, can afford the optimal bundle at the higher rent level, and the optimum utility level is attained with the new market rent profile,

$R^*(x)+c$. The market rent is thus indeterminate if $e=0$.

One important implication of this indeterminacy is that the optimal solution can be achieved without having a jump in the rent function even if we add the constraint that the market rent be equal to the rural rent at the boundary. After solving for the optimal allocation without the constraint, we simply lower the market rent curve until the rent at the boundary equals the rural rent. This observation yields the following proposition which is the result obtained by Solow and Vickrey (1971), and Kanemoto (1975).

Proposition 2: If the compensated demand for land is completely price inelastic, and if the market rent equals the rural rent at the edge of the city, then at the optimum the market benefit equals the market rent at the edge of the city and is greater in the rest of the city.

This proposition is illustrated in Figure 5. Note that the second best optimum coincides with the first best optimum, since, when demand for land is completely price inelastic, the only difference between them is the market rent that does not affect consumption decisions of households.

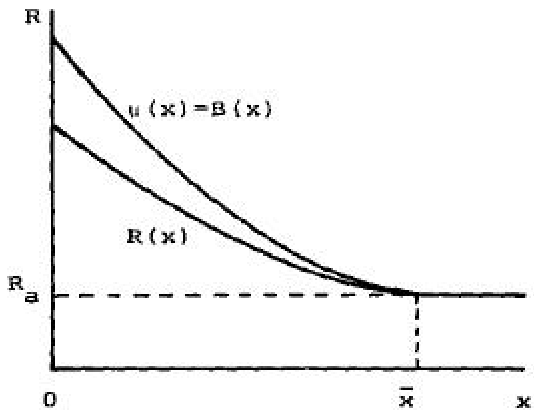

Figure 5
Completely Price Inelastic Case
with $R(\bar{x}) = R_a$

The proposition suggests that there is a strong tendency towards overinvestment in roads when $e=0$. Considering the results obtained in the preceding section, however, the proposition is somewhat misleading. As long as compensated demand for land is not completely price inelastic, the market rent is not indeterminate and we obtain a situation like the one depicted in Figure 1, where the social rent is higher than the market rent near the center and lower near the edge. Although the market benefit approaches the social rent as the elasticity tends to zero, the relationship between the market rent and the social rent remains basically the same as long as the elasticity is positive, since (2.26) is effective even when the elasticity is very small. How the relationship among the market rent, the social rent, and the market benefit changes as the elasticity becomes smaller is illustrated in Figure 6. If the elasticity is greater than

1, the market benefit is greater than the market rent at the edge of the city, as in Figure 6a (which reproduces Figure 4). If the elasticity is between 0 and 1, the market benefit falls below the market rent but is still higher than the social rent at $x = \bar{x}$, as in Figure 6b (or Figure 3). As the elasticity approaches zero, the market benefit tends to the social rent, but the market rent remains higher than the social rent at $x = \bar{x}$. In the limit we obtain the case, depicted in Figure 6c, in which the market benefit is less than the market rent near the edge of the city. Thus Figure 5 and hence Proposition 2 cannot approximate the case where the elasticity is close to, but not exactly, zero.

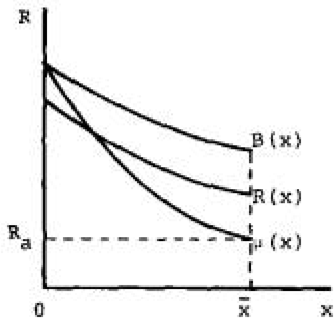

Figure 6a. e>1

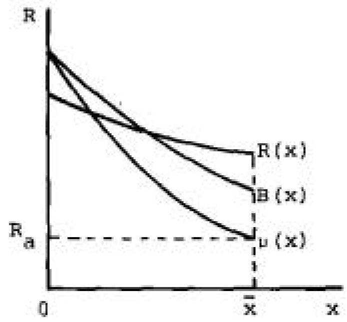

Figure 6b. 0<e<1

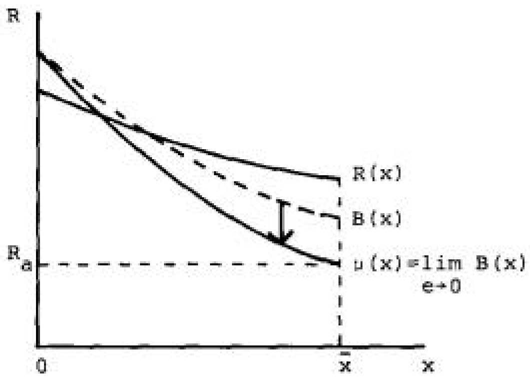

Figure 6c. e→0

Figure 6. Completely Price Inelastic Case as a Limit as e → 0.

The conclusion that the naive benefit-cost criterion has a tendency toward overinvestment is nevertheless correct, since the market city has a wider road than the optimum city, as shown in Kanemoto (1975). The main reason is that at the second best optimum the market rent is higher than the rural rent at the edge of the city. This tends to make the market city larger than the second best city. In the models in Solow and Vickrey (1971) and Kanemoto (1975), where a fixed amount of land is required for nontransportation use, the city can grow only if the road is widened.

3. An Open City

Next, consider an open and small city in which the utility level is given from outside: $u = \bar{u}$. This time we consider the, absentee-landlord case. The income of a household is given by the value of marginal productivity of labour: $y = w$. These two conditions replace the population constraint (2.1) and the resource constraint (2.2) in a closed city.

The bid rent function (1.4) and the compensated demand functions, (1.7) and (1.8), become

$$R(x) = R(w - t(x), \bar{u}), \tag{3.1}$$

$$z(x) = z(R(x), \bar{u}), \tag{3.2}$$

$$h(x) = h(R(x), \bar{u}). \tag{3.3}$$

The net product of the city after the cost of maintaining the given utility level of residents,

$$\int_0^{\bar{x}} \{[w - z(x) - t(x)]N(x) - R_a q(x)\} dx, \tag{3.4}$$

is maximized. The Hamiltonian and the Lagrangian for this problem are

$$\begin{aligned}\Phi = & \frac{w - z[R(w - t(x), \bar{u}), \bar{u}] - t(x) - l(x)}{h[R(w - t(x), \bar{u}), \bar{u}]} L_H(x) - R_a q(x) \\ & + h(x) g(T(x), L_T(x))\end{aligned} \tag{3.5}$$

and

$$\Psi = \Phi + m(x)[q(x) - L_H(x) L_T(x)], \tag{3.6}$$

where $l(x)$ and $h(x)$ are respectively the adjoint variables associated with (1.11) and (1.1), and $m(x)$ is a Lagrange multiplier for (1.13).

The control variables are $L_H(x)$ and $L_T(x)$, and the control parameters are $\bar{x}$, $t(\bar{x})$ and $T(0)$. We assume that a city planner can determine the boundary of the city regardless of the level of the market rent there. Under this assumption there is no constraint on $t(\bar{x})$.

The first order conditions are

$$m(x) = R(x) - l(x)/h(x), \tag{3.7}$$

$$m(x) = B(x) - g_L(T, L_T) \int_x^{\bar{x}} \frac{m - R}{R} eN dx', \tag{3.8}$$

$$l'(x) = -m g_T / g_{L'} \tag{3.9}$$

$$l(0) = 0, \tag{3.10}$$

$$m(\bar{x}) = R_a, \tag{3.11}$$

where $B(x)$ is defined by (2.24), $m(x)$ satisfies (2.10), and e is the price elasticity of compensated demand for land as defined by (2.18). These conditions are similar to those obtained for a closed city and have similar interpretations.[6]

Calculations of the correct benefit and cost are the same as in the closed city except for the boundary conditions. From (3.7) through (3.10), the difference between the shadow rent and the market rent, $r(x)$, satisfies the differential equation

$$r'(x)h(x)+r(x)h'(x)=\left[T+\int_x^{\bar{x}}(r/R)eNdx'\right]g_T(T,L_T), \tag{3.12}$$

with the boundary condition

$$r(0)=0.$$

When this differential equation is solved, the social marginal cost of the road is given by $r(x)+R(x)$, and the social marginal benefit is

$$B(x)-g_L\int_x^{\bar{x}}(r/R)eNdx'.$$

Next, we compare the market benefit, $B(x)$, and the market rent, $R(x)$, at the second best optimum to see whether the naive benefit-cost criterion results in overinvestment in roads. In order to do so, we first compare the market rent, $R(x)$, and the social rent, $m(x)$. Since congestion tolls are not imposed, the social transportation costs are greater than the private trans-portation costs. The social rent, therefore, tends to be steeper than the market rent. In the open city, however, both rents are equal at the center by the transversality condition (3.10). Thus the social rent is lower than the market rent everywhere in the city except at the center where they are equal, and the following theorem is obtained.

Theorem 3:

$$m(0)=R(0),$$

and

$$m(x)<R(x), \qquad 0<x\leq\bar{x}.$$

We omit the proof, which is quite simple. Notice that this theorem holds even if the compensated demand for land is completely price inelastic.

Next, we compare the market benefit and the social rent. The market benefit differs from the social rent by the indirect effect through consumption decisions. A reduction in transportation costs at a radius has a tendency to raise the market rent beyond that radius. Since, by Theorem 3, the market rent is higher than the social rent, raising the market rent increases the gap. The indirect effect of a reduction in transportation costs thus causes a social loss, and the social benefit is smaller than the

[6] As in the closed city, $h(x)$ may become positive, and a jump in $t(x)$ may occur. However, the following theorems and proposition hold even if $t(x)$ has a jump.

market benefit.

Theorem 4: If e > 0 for all x, then

$$m(x) < B(x), \qquad 0 \leq x \leq \bar{x}.$$

For $x < \bar{x}$, the Theorem is immediately obtained from (3.8) and Theorem 3. At $x = \bar{x}$, $L'H\hat{o}pital's$ rule yields the inequality as in the proof of Theorem 2.

The above two theorems show that the market benefit is greater than the market rent at least near the center. The naive benefit-cost analysis, therefore, has a tendency to overinvest in roads near the center. The following proposition shows that this pattern is reversed near the edge of the city if the elasticity of demand for land is less than one.

Proposition 3: Suppose the compensated demand for land is not completely price inelastic. Then the market benefit is greater than the market rent near the center. If, further, the price elasticity of compensated demand for land is less (greater) than one, the market benefit is smaller (greater) than the market rent near the edge of the city.

The proof is the same as that of Proposition 1. Figure 7 depicts the case of inelastic demand. Figure 8 the case of elastic demand. Notice that relative positions of the market benefit and the market rent are the same as in a closed city though their relationships with the social rent are different.

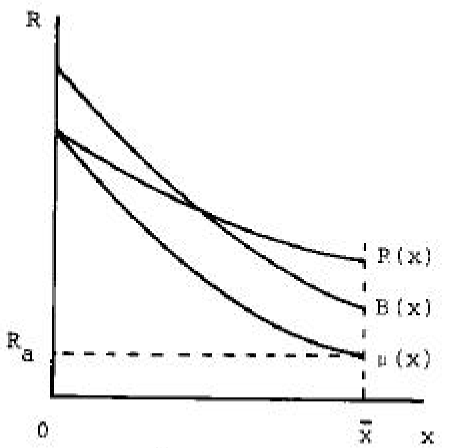

Figure 7
Inelastic Demand:
An Open City

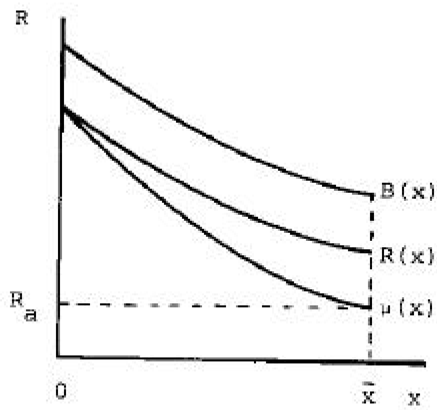

Figure 8
Elastic Demand:
An Open City

In a closed city the market benefit equaled the social rent at the center, but in an open city the market benefit exceeds the social rent everywhere. In a closed city the market rent crossed the social rent at some intermediate point, while in an open city the market rent is equal to the social rent at the center.

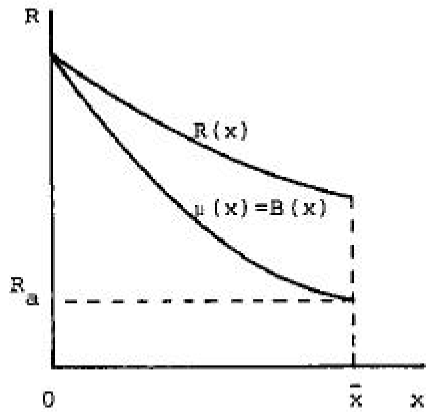

Figure 9
Completely Price Inelastic Demand:
An Open City

When compensated demand for land is completely price inelastic, the second term on the RHS of (3.8) vanishes. The market benefit, therefore, coincides with the social rent and we obtain the following proposition which is illustrated in Figure 9.

Proposition 4: If compensated demand for land is completely price inelastic, then the market benefit is equal to the market rent at the center and is smaller than the market rent in the rest of the city.

Thus, in sharp contrast to Proposition 2 in a closed city, there is a tendency to underinvest in roads everywhere in the city. Since the market rent is higher than the rural rent at the edge of the city, however, the market city tends to be bigger than the optimum city. This increases the total population of the city and hence the total traffic, which works in the direction of widening the road. In Kanemoto (1975), the road is shown to be wider in the market city than in the optimum city.

4. An Economy with Many Cities

In this section we consider an economy consisting of many cities. The model is the same as that in section 4 of the preceding chapter. The population constraint is

$$P = nP_c, \tag{4.1}$$

where P, P_c, and n are respectively the population of the economy, the population of a city, and the number of cities. The resource constraint is

$$\int_0^{\bar{x}} \left\{ \frac{z\left[R(y-t(x),u),u\right]+t(x)}{h\left[R(y-t(x),u),u\right]} + R_a q(x) \right\} dx \le F(P_c)\,. \tag{4.2}$$

The aggregate production function, $F(P_c)$, has increasing returns to scale. The boundary condition for $T(x)$ at $x = 0$ is

$$T(0) = P_c\,. \tag{4.3}$$

The common utility level is maximized under the constraints (1.1), (1.2), (1.11), (1.12), (1.13), (4.1), (4.2), and (4.3). The control variables are $L_H(x)$ and $L_T(x)$, and the control parameters are P_c, n, y, u, $\bar{x}$, and $t(\bar{x})$. The Hamiltonian for this problem is

$$\Phi = -l(x)\frac{L_H(x)}{h[R(y-t(x),u),u]} + h(x)g(T(x), L_T(x))$$

$$-d\left\{\frac{z[R(y-t(x),u),u]+t(x)}{h[R(y-t(x),u),u]}L_H(x) + R_a q(x)\right\}, \tag{4 4}$$

and the Lagrangian is

$$\Psi = \Phi + m(x)[q(x) - L_H(x) - L_T(x)], \tag{4.5}$$

where $l(x)$, $h(x)$, and d are adjoint variables associated with (1.11), (1.1), and (4.2) respectively, and $m(x)$ is a Lagrange multiplier for (1.13).

After dividing $l(x) - l(0)$, $h(x)$, and $m(x)$ by d and denoting the obtained variables by $l(x)$, $h(x)$, and $m(x)$ respectively, the first order conditions become (2.19), (2.23), (2.25), (2.26),

and

$$l(x) = -\int_0^x h(x')g_T(T(x'), L_T(x'))dx'\,, \tag{4.6}$$

$$\frac{1}{h(x)}[F'(P_c) - z(x) - t(x) - l(x)] = m(x)\,, \tag{4.7}$$

$$\int_0^{\bar{x}} \frac{1}{V_I} N dx - \frac{1}{d} = \int_0^{\bar{x}} [m(x) - R(x)]\frac{\hat{h}_R}{h}\frac{N}{V_I}dx\,. \tag{4.8}$$

(4.6), (4.7), and (4.8) correspond to (2.6), (2.20), and (2.27). As before, $-h(x)$ is the social cost of increasing commuting costs of all households passing through x by one unit. $-h(x)g_T$ is, therefore, the social cost of an increase in congestion caused by a unit increase in the traffic at x, and $l(x)$ is the social congestion costs that a resident at x imposes on other travelers by commuting from x to the center.

Multiplying (4.7) by $h(x)N(x)$ and integrating from 0 to $\bar{x}$ yields

$$P_c F'(P_c) = \int_0^{\bar{x}} [(z+t)N + lN + mL_H]dx\,.$$

Comparing this equation with the resource constraint (4.2) and noting that the resource constraint holds with equality at the optimum, we obtain

$$-\left[F(P_c)-P_cF'(P_c)\right]=\int_0^{\bar{x}}\left[lN+mL_H-R_aq\right]dx\,. \qquad (4.9)$$

Thus *the operating loss of a producer at the optimum equals the total social congestion costs, plus the total social rent of residential land, minus the total payment of the rural rent*. This is similar to the result obtained in the previous chapter: the operating loss of a firm equals the total congestion tolls, plus the total rent of residential land, minus the total payment of the rural rent. The difference is that there are no tolls capturing the social congestion costs in this chapter and the social rent does not equal the market rent. It is quite natural that the same relationship holds for social values instead of market values.

As shown in subsection 2.1, if we assume constant returns to scale in transportation technology, the social congestion costs equal the total shadow rent of roads at each radius:

$$l'(x)T(x)=m(x)L_T(x), \qquad 0\le x\le \bar{x}\,.$$

Then by integration by parts, (4.9) becomes

$$-\left[F(P_c)-P_cF'(P_c)\right]=\int_0^{\bar{x}}\left[m(x)-R_a\right]q(x)dx\,. \qquad (4.10)$$

This is again similar to the relationship obtained in Chapter IV. The operating loss of a producer equals the difference between the total social rent and the total payment of the rural rent, where the total social rent includes the rent on the road. Note that this relationship does not in general hold for the market rent, since (2.26) requires that the sums of the market and social rents be equal when they are weighted by eN/R which equals $q(x)$ only by chance.

It is easy to see that the social benefit and cost can be calculated exactly in the same way as in the closed city. The relationships among the social rent, the market rent, and the market benefit are also the same as in the closed city.

5. Concluding Remarks

The analysis in this and the preceding chapters are centered on the interaction between pricing of traffic congestion and the investment decision of roads. If congestion is optimally priced, the investment decision is quite straightforward. The allocation of land between roads and residence must be determined in such a way that the marginal *social* benefits of widening the road equals the marginal social cost at each radius. The marginal social benefit at a radius is simply the marginal direct saving in transportation costs with the volume of traffic there fixed; the marginal social cost is the market rent of the residential land.

This simplicity in the benefit-cost criterion is the general property of the first best world where all goods are priced properly. Since all prices reflect the true social marginal values of the goods, prices may stand in for social values in the calculation of benefits and costs. Thus the marginal social cost of widening the road is given by the market rent in our model.

The fact that all prices reflect the social marginal values has another important implication. When the road is widened, commuting costs decrease and hence the land rent rises. This induces a change in the allocation of the entire city through a change in the consumption bundles of households. The change however, can be ignored in the calculation of the marginal benefit and cost. The reason is that the *social* values of the induced change is zero, since the *market* value of the induced change is zero due to the envelope property, and the market value equals the social value when all prices equal the social marginal values. This is the reason why the marginal social benefit equals the marginal direct saving in transportation costs with the fixed traffic volume.

The simplicity disappears if traffic congestion is not properly priced. Prices no longer reflect the marginal social values of goods accurately, and in particular, the market rent does not equal the social marginal value of residential land. Accordingly, the cost side of the benefit-cost criterion must be changed. The benefit side also becomes more complicated since the induced change in the consumption decisions has a nonzero social value or loss. The naive benefit-cost analysis usually adopted by policy makers, therefore, gives rise to an inefficient land use.

Unfortunately, the correct benefit-cost criterion is difficult to calculate. Furthermore, boundary conditions that must be used to calculate the benefit-cost criterion are different between closed and open cities. The correct benefit cost criterion is, therefore, unlikely to be practical, at least until we know more. Meanwhile, it would be useful to know whether the naive benefit-cost analysis leads to too wide a road.

The results in Chapter IV suggest that the road in the city with the naive benefit-cost analysis is usually wider than that it in the first best optimum where congestion tolls are levied and roads are optimally built. This comparison, however, may not be useful, since it is difficult to levy congestion tolls because of very high administrative costs. The analysis in this chapter is a partial attempt at the comparison with the second best optimum in which roads are built optimally under the constraint that congestion tolls are impossible. We compared the benefit and the cost in the erroneous benefit-cost criterion at the second best optimum and showed that the benefit exceeds the cost near the center and that the benefit exceeds the cost also near the edge in the case of price elastic demand for land and is less than the cost in the price inelastic case. This implies that, if the erroneous benefit-cost criterion is adopted only in a very narrow ring near the center, overinvestment in roads will result. If it is adopted near the edge underinvestment will result in the inelastic case and overinvestment in the elastic case.

Unfortunately, the analysis is not conclusive if the erroneous benefit-cost criterion is adopted everywhere in the city. It seems, however, more likely that the naive benefit-cost criterion leads to overinvestment in roads. The major reason is that the market rent is higher than the rural rent at the second best optimum and the market city with the benefit-cost criterion tends to be bigger, which is made possible only by building wider roads and lowering commuting costs. The results obtained in somewhat different models by Wheaton (1978), Pines and Sadka (1979), and Wan (1979) also support this conjecture.

Notes

The analysis in this chapter originates in Solow and Vickrey (1971). They formulated a transportation cost minimization problem in a long narrow city framework and asked the question whether or not the cost-benefit analysis based on the market rent yields too wide a road. To see this, they compared the benefit from widening the road with the market rent at the optimum configuration.

They, in effect, made the following three assumptions. First, the city was assumed to be closed in the sense that the total production (or the total population when interpreted as a residential model) in the city was fixed. Second, they assumed that only production required land, that production required only land, and that the price elasticity of demand for land was zero so that demand for space was not affected by the level of land rent. Third, the market rent was constrained to be equal to the rural rent (in their case, zero rent) at the boundary of the city. Their model, therefore, corresponds to the case of subsection 2.3 in this chapter. Naturally, they obtained exactly the same conclusion as in Proposition 2 - that the benefit is greater than the market rent everywhere in the city - and concluded that the cost-benefit analysis based on market rent has a tendency to overinvest in roads.

Kanemoto (1975) introduced an open city facing a given export price, and compared it with a closed city. The model is essentially the same as the completely-price-inelastic case of the open city in this chapter. The relationship between the market benefit and the market rent at the optimum allocation of land is the same as that in Proposition 4.

Since these models assume completely price inelastic demand for land, the first best allocation coincides with the second best allocation. The second best allocation differs from the first best allocation if substitution between land and other goods is possible. Solow (1975) first considered this type of a second best problem in a spatial equilibrium framework. He maximized the utility level of households within the class of linear road width functions in a closed city. According to his numerical calculations, the market benefit from widening the road is greater than the market rent. He explained this result as follows. Since congestion tolls are not levied, the market rent is flatter than the social rent. But the two rents are equal to the rural rent at the edge of the city. The market rent is therefore lower than the social rent, and the value of land is underestimated in the naive cost-benefit calculations.

Our analysis indicates that this explanation fails to notice the following two aspects of the second best allocation. First, though the social rent is steeper than the market rent, the two are not in general equal at the edge of the city. Our analysis shows that the market rent is higher than the social rent at the edge of the city. Second, the market benefit from widening the road does not correctly reflect the social benefit. The market benefit is greater than the social benefit because the adjustment of consumption caused by a decrease in transportation costs involves social costs when congestion tolls are not levied.

Kanemoto (1976) considered a production city with substitutability between labour and land in an open city framework. The results are parallel to those in section 3. The analysis of a closed city is based on Kanemoto (1977a).

Wheaton (1978) considered a similar problem in a nonspatial framework with more than one type of roads. He also analyzed the problem of finding the optimal uniform congestion tax which is constrained to have the same tax rate on all roads regardless of different degrees of congestion.

Arnott (1979) extended our analysis to the case where the road is of arbitrary width. Arnott and MacKinnon (1978) obtained the numerical solution of using the fixed point algorithm. Wan (1979) applied the perturbation method to the second best problem and also obtained numerical solutions.

Pines and Sadka (1979) considered a discrete model in which a city is divided into two rings. Assuming that the areas of the two rings are fixed, they showed that there is more investment in roads in the market city with the naive benefit-cost analysis than in the second best city.

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