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AGGLOMERATION ECONOMIES AND A TEST FOR OPTIMAL CITY SIZES IN JAPAN*

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Abstract:

We estimate aggregate production functions for metropolitan areas in Japan to derive the magnitudes of agglomeration economies. Using the estimates of agglomeration economies, we test if Japanese cities, in particular, Tokyo, are too large.

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1. Introduction

The purpose of this article is two-fold. First, we estimate aggregate production functions for metropolitan areas in Japan to derive the magnitudes of agglomeration economies. Second, we use the estimates of agglomeration economies to test if Japanese cities, in particular, Tokyo, are too large.

A number of economists have estimated agglomeration economies for Japanese cities, but their estimates are limited to manufacturing industries.[1] Service industries have become dominant in larger cities and focusing only on manufacturing industries may cause serious biases. Furthermore, they used either jurisdictional data or population density as a surrogate for city size because data for metropolitan areas were not available. Unlike in the U.S. where data for Standard Metropolitan Statistical Areas are available, the Japanese government provides data only for legal jurisdictions. In this article, we construct our own metropolitan data set to estimate the magnitudes of agglomeration economies for Japanese cities.

Another focus of this paper is to examine whether cities in Japan are too large. Currently, the Parliament is discussing a plan to move the capital out of Tokyo and the main reason for this movement is the perception that the concentration of economic activities in Tokyo is excessive. It is certainly true that the population of the Tokyo metropolitan area exceeds 30 million and congestion in commuter railways is almost unbearable. However, it is also true that Tokyo is very convenient for conducting businesses because almost everyone a business-person wants to talk to is located in downtown Tokyo. In order to check whether Tokyo is too large or not, we have to compare the agglomeration economies with a variety of deglomeration economies such as longer commuting time and congestion externalities.

It is well known in urban economics that the optimal city size satisfies the Henry George Theorem.[2] For example, if the only agglomeration forces are externalities among firms in a city and the only deglomeration forces are commuting costs of workers who work at the center of the city, then the optimal city size is achieved when the Pigouvian subsidy for the agglomeration externalities equals the total differential urban rent.[3] As argued in Kanemoto (1980), however, there is no reason to believe that the optimal city size is attained in equilibrium. Because of difficulty in forming a new agglomeration, we have multiple equilibria and furthermore the equilibrium city sizes tend to be too large.

Unfortunately, a direct test of the Henry George Theorem is very difficult because good land rent data are not available and we have to rely on land price data instead. The conversion of land prices into land rents is bound to be inaccurate in Japan where the price/rent ratio is extremely high and has fluctuated enormously.

Instead of testing the Henry George Theorem directly, we compute the ratio between the total land value and the total Pigouvian subsidy for each metropolitan area and see if there is a significant difference between different levels of hierarchy of cities. Our hypothesis is that cities form a hierarchical structure where Tokyo is the only city at the top. Equilibrium city sizes tend to be too large at each level of hierarchy. Divergence from the optimal size cannot differ much between cities at the same hierarchical level because otherwise utility

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[1] See Kawashima (1975), Nakamura (1985), and Tabuchi (1986), for example.

[2] The Henry George Theorem is obtained by Arnott and Stiglitz (1979), Henderson (1977), and Kanemoto (1980) among others.

[3] The differential urban rent is the urban rent minus the rural rent at the edge of the city.

levels differ significantly across cities and inter-city migration occurs. The divergence from the optimum could however be significantly different between different levels of hierarchy. At a low level of hierarchy the divergence tends to be small because it is relatively easy to add a new city when existing cities are too large. At a higher level it becomes more difficult to create a new city because larger agglomerations are more difficult to form. We therefore test if the divergence from the optimum is larger for larger cities, in particular, if the ratio between the total land value and the total Pigouvian subsidy is significantly larger for Tokyo than for other cities.

The organization of this paper is as follows. In Section 2, we estimate aggregate production functions for metropolitan areas to obtain quantitative estimates of agglomeration economies. In Section 3, we compare agglomeration economies with the total land value. Section 4 summarizes the results in the paper and discusses possibilities for elaboration and extension.

2. Estimation of Agglomeration Economies

In this section we estimate aggregate production functions for metropolitan areas in Japan. The first task is to define metropolitan areas. For each of the metropolitan areas, we construct data for capital and labor inputs, value added, and social overhead capital. Using this data set, we estimate production functions for metropolitan areas to obtain estimates of agglomeration economies.

2.1. The Definition of a Metropolitan Area

Because the Japanese government does not publish data for metropolitan areas unlike in the United States where a variety of economic data are available for Standard Metropolitan Statistical Areas (SMSAs). Our first (laborious) task is to construct data for metropolitan areas.

Although there is no official definition of metropolitan areas, a number of researchers have developed their own definitions. As far as we know, there are three: the SMEA (Standard Metropolitan Employment Area) by Hiroyuki Yamada and Kazuyuki Tokuoka, the Functional Urban Core (FUC) by Tatsuhiko Kawashima, and the Integrated Metropolitan Area (IMA) by Shogo Takeuchi. We use the last one because it was readily available when we started our research.

The IMAs are defined through the following iterative procedure.[4]

First, we combine a municipality (city, town, or village) with another if doing so increases the ratio of internal employment (i.e., the proportion of residents in an urban area who work within the area). That is, if the ratio of internal employment in the combined area is higher than those in the two municipalities, then we integrate the two municipalities to form a new larger urban area. This condition is checked for each municipality with its three most closely related municipalities (i.e., three municipalities that employ largest numbers of residents in the municipality). If there are more than one municipality that satisfies the condition, we choose the one with the highest internal employment ratio after integration. A municipality whose internal employment ratio is higher than 90% is not, however, integrated with another municipality.

This integration procedure is carried out for all municipalities, and the joins of all combinations form urban areas. This completes the first iteration, and we repeat the same procedure starting with the newly defined urban areas. The iteration stops when no more new integration occurs.

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[4] See Suzuki and Takeuchi (1994) for more detailed explanations of the IMA.

The advantage of the IMA over other definitions that are similar to the SMSA in the U.S. is that it captures the degree of integration in relative terms. Because of this feature, IMAs can be defined in areas with low population densities if there exist significant commuting interactions between different municipalities. Compared with SMEAs and FUCs, IMAs tend to be larger. For example, the population of the Tokyo metropolitan area is 33,529,313 according to the IMA and 27,187,116 according to the SMEA in 1990.

2.2. Construction of IMA-based Data

In our estimation of agglomeration economies, we use cross-sectional data for IMAs in 1985. Although more recent data are available, we choose this year in order to avoid tumultuous changes in land prices in late 80's and early 90's. The variables that we use are production (i.e., value added), labor, private capital, and social overhead capital. Among these variables, only the labor force data (the number of workers at work place) are available for each municipality. Other data are available only at the prefecture level except for ten major cities.[5] We construct the IMA data from the prefectural data in the following manner.

First, the total production in each IMA is obtained by proportional allotment according to employment shares in manufacturing and non-manufacturing sectors. For example, take an IMA (denoted by A) that does not contain any of the major 10 cities. The numbers of workers in manufacturing and non-manufacturing sectors in the IMA are respectively $N(A,1)$ and $N(A,2)$. The prefecture (denoted by I) that contains the IMA has production (valued added), $Y(I,\ j)$ for $j=1,2$, and employment, $N(I,\ j)$ for $j=1,2$, for manufacturing and non-manufacturing industries. The total production in the IMA is then

[5] The ten major cities are Sapporo, Kawasaki, Yokohama, Nagoya, Kyoto, Osaka, Kobe, Hiroshima, Kitakyushu, and Fukuoka.

(1) $$Y(A) = \sum_{j=1}^{2} Y(I, j)\frac{N(A, j)}{N(I, j)}.$$

If the IMA contains one of the 10 major cities which have their own production data, the same proportional allotment procedure is applied to the part excluding the city.

Second, the private capital in each IMA is obtained by proportional allotment according to production shares in manufacturing and non-manufacturing industries (instead of employment shares).

Third, there are four types of social overhead capital and the allotment procedures differ slightly among them. The first type is social overhead capital for agriculture, forestry, and fishery. This type of social overhead capital is allocated according to the employment shares in the agricultural sector. The second type that represents the industrial infrastructure is allocated according to the production shares in the manufacturing industry. The third type is the capital stock in telecommunication and railway industries. This type is allocated according to total production (that includes both manufacturing and non-manufacturing industries). The fourth type is infrastructure for residents such as parks and neighborhood streets. This type is allocated according to population.

Sources of original data are

Number of workers at work place: 1985 Population Census

Population: 1985 Population Census

Production (Value Added) for manufacturing and non-manufacturing industries: Annual Report on Prefecture Accounts

Private Capital Stock: Ohkawara et. al. (1985) (Estimated by the CRIEPI).

Social Overhead Capital: Ohkawara et. al. (1985) (Estimated by the CRIEPI).

2.3. Production Functions with Agglomeration Economies

We estimate aggregate production functions for metropolitan areas to derive estimates of the magnitudes of urban agglomeration economies. An aggregate production function in a city is written as $Y = F(N, K, G)$, where *N*, *K*, *G*, and *Y* are respectively the employment, the private capital, the social overhead capital, and the total production (or value added) in a metropolitan area. We assume that in the absence of agglomeration economies the production function exhibits constant returns to scale with respect to labor and capital inputs. The degree of agglomeration economies can then be measured by the degree of increasing returns to scale of the estimated production function.

This approach can be justified if we assume that technological externalities exist between firms in a metropolitan area. For example, suppose a firm in a city receives external benefits from urban agglomeration, measured by the total employment *N*, and social overhead capital *G*. Assuming that the firm uses labor *n* and (private) capital *k* as inputs, we can write its production function as $f(n, k, N, G)$. For expositional simplicity, we assume that all firms are identical. The total production in a metropolitan area is then $Y = mf(N/m, K/m, N, G)$, where *m* is the number of firms in a metropolitan area. Free entry of firms guarantees that the size of an individual firm is determined such that the production function of an individual firm $f(n, k, N, G)$ exhibits constant returns to scale with respect to *n* and *k*. This condition determines the number of firms *m* as a function of other variables, $m = m^*(N, K, G)$. The aggregate production function is then

$$(2) \qquad F(N,K,G) = m^*(N,K,G) f\left(\frac{N}{m^*(N,K,G)}, \frac{K}{m^*(N,K,G)}, N, G\right).$$

This aggregate production function satisfies

$$F_N(N,K,G) = m\left[\frac{1}{m} f_n + f_N\right] + m_N^*\left[f - nf_n - kf_k\right], \qquad (3)$$
$$= f_n(n,k,N,G) + mf_N(n,k,N,G)$$

where subscripts denote partial derivatives and the second square bracket equals zero because of the constant-returns-to-scale condition mentioned above. The last term mf_N measures the marginal benefits of urban agglomeration economies.

The above model of technological externalities is attractive for its simplicity but as argued by Kanemoto (1990) it is difficult to believe that non-market interactions between firms are strong enough to create large metropolitan areas. A number of papers, e.g., Kanemoto (1990) and Krugman (1991), proposed an alternative approach relying on heterogeneity of final and/or intermediate products. They showed that, if the heterogeneity is combined with transportation and communication costs, agglomeration economies emerge even in the absence of technological externalities. Our aggregate production function may be interpreted as being derived from such a model. The normative properties of the heterogeneous good models have not been fully investigated, however, and it is not clear if our test for optimal city size in section 3 is valid also in these models. We take up this issue again in the conclusion section.

Although a variety of functional forms are possible for the urban production function, we first start with a simple Cobb-Douglas type:

$$Y = AK^{\alpha} N^{\beta} G^{\gamma}. \qquad (4)$$

Because we assume that an individual firm's production function has constant returns to scale with respect to labor and capital, the magnitude of urban agglomeration economies can be measured by the degree of scale economy, $\alpha + \beta - 1$.

It turns out that in our estimation the coefficient for the social overhead capital γ is either negative or statistically insignificant if we use the Cobb-Douglas production function. We therefore try another form:

(5) $$Y = AK^{\alpha} N^{1-\alpha} N^{\gamma \ln G}.$$

In this formulation the degree of agglomeration economies, $\gamma \ln G$, is increasing in the social overhead capital.

2.4. Estimation Results

We first estimate the simple Cobb-Douglas production function,

(6) $$\ln(Y/N) = A_0 + a_1 \ln(K/N) + a_2 \ln N + a_3 \ln(G/N),$$

where *Y*, *K*, *N*, and *G* are respectively the value added, private capital stock, employment, and social overhead capital in an IMA. The relationships between the estimated parameters and coefficients in the Cobb-Douglas production function (4) are $\alpha = a_1$, $\beta = a_2 + 1 - a_1 - a_3$, $\gamma = a_3$.

Table 1 reports the estimation results for equation (6) for different size groups. The coefficient for the social overhead capital is significantly negative for small IMAs with less than 200,000 residents and insignificant for larger size groups. If these estimates were correct, the marginal productivity of social overhead capital would be negative for small cities and zero for medium to large cities. It is however difficult to believe that the social overhead capital has negative impacts on production.

Table 1 Estimates of a Cobb-Douglas Production Function with Social Overhead Capital

Parameter	All IMAs (456)	Over 1 Mil. (17)	0.4 - 1 Mil. (34)	2-0.4 Mil. (32)	Under 0.2 Mil. (373)
A_0	0.66	-0.69	0.62	-2.50	0.74
	(6.47)	(-1.54)	(0.88)	(-1.55)	(5.89)
a_1	0.57	0.72	0.25	0.58	0.60
	(12.40)	(6.30)	(4.07)	(6.68)	(10.11)
a_2	0.01	0.07	0.05	0.24	0.00
	(3.08)	(2.75)	(0.96)	(1.87)	(0.06)
a_3	-0.26	-0.05	-0.08	-0.02	-0.27
	(-9.06)	(-0.32)	(-1.00)	(-0.15)	(-8.42)
$\overline{R}^2$	0.40	0.81	0.45	0.63	0.29

Note: Numbers in parentheses in the second row are numbers of samples, and those in other rows are t ratios.

Table 2 Estimates of a Cobb-Douglas Production Function without Social Overhead Capital

Parameter	All IMAs (456)	Over 1 Mil. (17)	0.4 - 1 Mil. (34)	0.2-0.4 Mil. (32)	Under 0.2 Mil. (373)
A_0	0.19	-0.77	0.19	-2.58	0.37
	(1.95)	(-2.13)	(0.34)	(-1.73)	(2.91)
a_1	0.49	0.72	0.25	0.59	0.47
	(10.02)	(6.53)	(4.05)	(6.81)	(7.54)
a_2	0.03	0.07	0.07	0.25	0.01
	(6.60)	(2.93)	(1.58)	(1.98)	(1.69)
$\overline{R}^2$	0.30	0.81	0.43	0.63	0.15

Note: Numbers in parentheses in the second row are numbers of samples, and those in other rows are t ratios.

Because the coefficient for the social overhead capital is either insignificant or negative, we omit the variable for the estimation of the Cobb-Douglas form. The results are shown in Table 2. The coefficient in which we are most interested is $a_2 = \alpha + \beta - 1$ which measures the degree of increasing returns to scale in urban production. Assuming that each firm's production function exhibits constant returns to scale with respect to capital and labor inputs, we interpret the scale economy of the urban aggregate production function as representing agglomeration economies.

Table 2 shows that agglomeration economies are statistically significant for all size groups although they are small in IMAs with population less than 200,000. In this size group doubling the city size increases production only by 1%. In the two size groups of over 400,000 inhabitants, the production increases are about 7%. It is surprising that in the medium size group (cities with 200,000 to 400,000 residents) agglomeration economies are very large at about 25%.

Our result that the estimated coefficient for social overhead capital is negative in Table 1 may be due to non-linearity in the way in which social overhead capital works. We therefore estimate a modified Cobb-Douglas form (5). In this form, productivity improvements by social overhead capital get larger as the employment in the city increases. Table 3 reports the estimation results for a logarithmic transformation of equation (5):

$$\ln(Y/N) = A_0 + a_1 \ln(K/N) + a_2 \ln N \ln G \,, \tag{7}$$

where the parameters satisfy $a_1 = \alpha$ and $a_2 = \gamma$.

The last row in Table 3 shows the degree of increasing returns to scale (with respect to *K* and *N*). Because of non-linearity, the degree depends on the size of the city. The numbers in Table 3 evaluate the degrees of scale economy at the size of Tokyo IMA. The degrees of scale economy are considerably smaller in Table 3 than those in Table 2.

Table 3 Estimated Parameters for a Modified Cobb-Douglas Production Function

Parameter	All IMAs (456)	Over 1 Mil. (17)	0.4 - 1 Mil. (34)	0.2-0.4 Mil. (32)	Under 0.2 Mil. (373)
A_0	0.31	-0.30	0.67	-0.95	0.42
	(3.36)	(-1.14)	(1.99)	(-1.16)	(3.43)
a_1	0.48	0.72	0.25	0.59	0.47
	(9.87)	(6.48)	(4.09)	(6.70)	(7.52)
a_2	0.0014	0.0022	0.0023	0.0081	0.0006
	(6.89)	(2.89)	(1.21)	(1.63)	(1.61)
$\overline{R}^2$	0.30	0.81	0.41	0.62	0.15
Scale Economy	0.026	0.040	0.042	0.146	0.010

Note: Numbers in parentheses in the second row are numbers of samples, and those in other rows are t ratios.

Another explanation for a negative coefficient for social overhead capital is that infrastructure investment has been used for redistribution across regions. That is, relatively more investment has been allocated to rural regions whose average incomes are lower. Because of this tendency, less productive cities have relatively more social overhead capital and a simple cross-section regression yields a negative coefficient. This can be interpreted as an example of a simultaneous equation bias where the supply side of social overhead capital is mixed up with the demand (or productivity) side. We have tried to find instrumental variables that remove the bias but so far we have not been successful.

3. A Test for Optimal City Sizes

Using the estimates of agglomeration economies obtained in the preceding section, we examine whether or not the cities in Japan (especially Tokyo) are too large.

3.1. The Henry George Theorem for the Optimal City Size

The so-called Henry George Theorem obtained by a number of urban economists in 1970's characterizes conditions for the optimal city size.[6] Depending on the sources of agglomeration economies and counteracting deglomeration economies, the theorem takes different forms. In all cases, however, the optimal city size is obtained when a certain measure of agglomeration benefits equals the total differential urban rent in the city, where the differential urban rent means the difference between land rent in an urban area and the rural rent at the edge of the city.

In this paper we have assumed two potential sources for agglomeration economies, i.e., externalities between firms in a city and social overhead capital. Deglomeration economies are caused by scarcity of space in the sense that, as a city expands in size, the average commuting distance increases. If we assume that all residents are homogeneous, the optimal city size that maximizes the utility level of a resident satisfies the following version of the Henry George Theorem.

Let us first consider the estimates in Table 2 that ignore social overhead capital. The optimal city size then requires that the total differential urban rent in the city equal the total Pigouvian subsidy that must be given to the sources of agglomeration economies.[7] In our formulation, an individual firm's production function can be written as

$$f(k,n,N) = Ak^{a} n^{1-a} N^{a+b-1},$$

where k and n are respectively capital and labor inputs and N is the total work force in the city which represents agglomeration economies. Because the Pigouvian subsidy per

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[6] For example, see Arnott and Stiglitz (1979), Henderson (1977), and Kanemoto (1980).

[7] See Kanemoto (1980), Chapter 2.

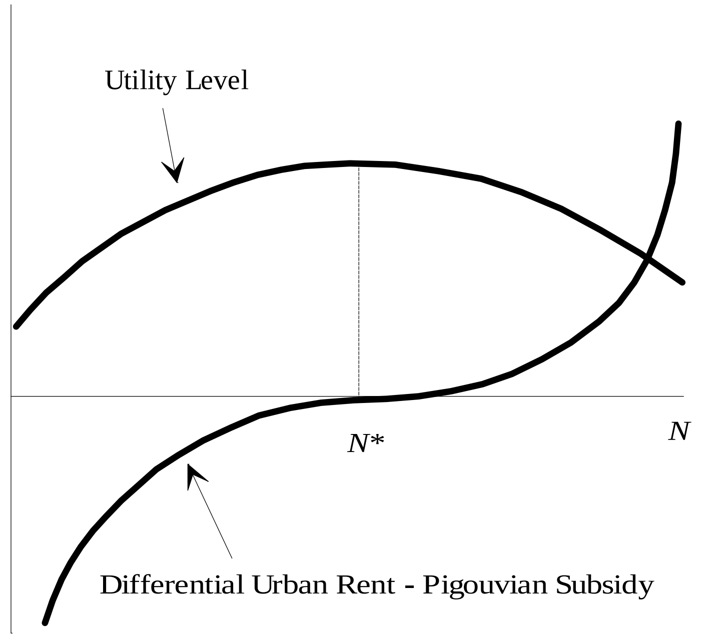
employer is $m\partial f / \partial N$, the total Pigouvian subsidy in a city is

$$PS = Nm\partial f / \partial N = (\alpha + \beta - 1)Y, \tag{8}$$

where m is the number of firms in the city that satisfies $m = N / n$ and Y is the total production, $Y = AK^{a} N^{b}$. The Henry George Theorem states that, if the city size is optimal, the total Pigouvian subsidy equals the total differential urban rent in a city. Furthermore, it is easy to show that the second order condition for the optimum implies that the Pigouvian subsidy is smaller than the total differential rent if the city size exceeds the optimum as shown in the following figure. We may therefore conclude that the city is too large if the total differential rent exceeds the total Pigouvian subsidy.

Next, let us introduce social overhead capital. The condition for optimal city size depends on the degree of publicness of social overhead capital. In the case of a pure local public good, all residents in a city can jointly consume it without suffering from congestion. Most of the social overhead capital does involve considerable congestion and cannot be regarded as a pure local public good. If the social overhead capital is a pure local public good, then applying an analysis similar to Chapter 3 of Kanemoto (1980) shows that the agglomeration benefits that must be equated with the total differential urban rent are the sum of the Pigouvian subsidy and the cost of the social overhead capital. For impure local public goods, the agglomeration benefits include only part of the costs of the goods. In the following, we consider two extreme cases, i.e., pure local public good and pure private good cases.

Figure 1 The total differential urban rent and the Pigouvian subsidy



The estimated equation in Table 3 may be interpreted as being derived from an individual production function of the form $f(k,n,N,G) = Ak^{\alpha}n^{1-\alpha}N^{\gamma \ln G}$. For this equation the total Pigouvian subsidy in the city is

(9) $$PS = \gamma(\ln G)Y.$$

We compare this with the total urban rent or the total urban rent minus the social overhead capital.

Note that our definition of optimal city size can be interpreted in two ways. One interpretation is that it describes the result of maximizing the utility of residents in a particular city with respect to its size. The other interpretation is optimization with respect to the number of homogeneous cities. For example, if the population of a country is fixed and everyone in the country lives in one of many homogeneous cities, then optimizing with respect to the number of cities is equivalent to maximizing the utility of a city with respect to its population.[8]

3.2. A Hierarchy of Cities and a Test for Optimal City Sizes

In reality different cities have different mixtures of industries and serve different functions. If the cities form a hierarchy in which cities at a certain hierarchical level are homogeneous, however, the above argument can be applied to cities at each level. For example, Tokyo is too large if having more than two cities at the top of the hierarchy increases the welfare of residents. The Henry George Theorem shows that this is true when the total differential urban rent exceeds the total agglomeration benefits.

In theory the Henry George Theorem can be tested for each city, but a direct test is very difficult because good land rent data are not available and we have to use land price data instead. In order to obtain the total differential urban rent from our land price data, we have to make the following adjustments. First, we must subtract development costs from the land value to obtain the value of undeveloped urban land. Second, we compute the total land rent by multiplying this by an appropriate discount rate. Third, we have to subtract the rural rent from the total land rent to produce the total differential urban rent. It is difficult to obtain good estimates for the three key variables, i.e., land development costs, the discount rate, and the rural rent.

The estimation of the discount rate is particularly difficult because the price-rent ratio has been extremely high and has fluctuated enormously over time in Japan. For example, Table 4 shows that the ratio between the total land value and GNP in Japan moved up from 2.48 in 1970 to 5.35 in 1990 and then down to 4.01 in 1993.

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[8] Strictly speaking, the fact that the number of cities must be an integer causes complications.

Because of the difficulty in converting land prices into land rents, we do not try to test the Henry George Theorem directly. What we do instead is to compute the ratio between the total land value and the total Pigouvian subsidy for each metropolitan area and see if there is a significant difference between different levels of hierarchy. The following argument shows that there is a good reason to believe that cities tend to be too big and that the divergence from the optimum tends to be larger for cities at a higher level of hierarchy.

Table 4 The Ratio Between the Total Land Value and GNP in Japan

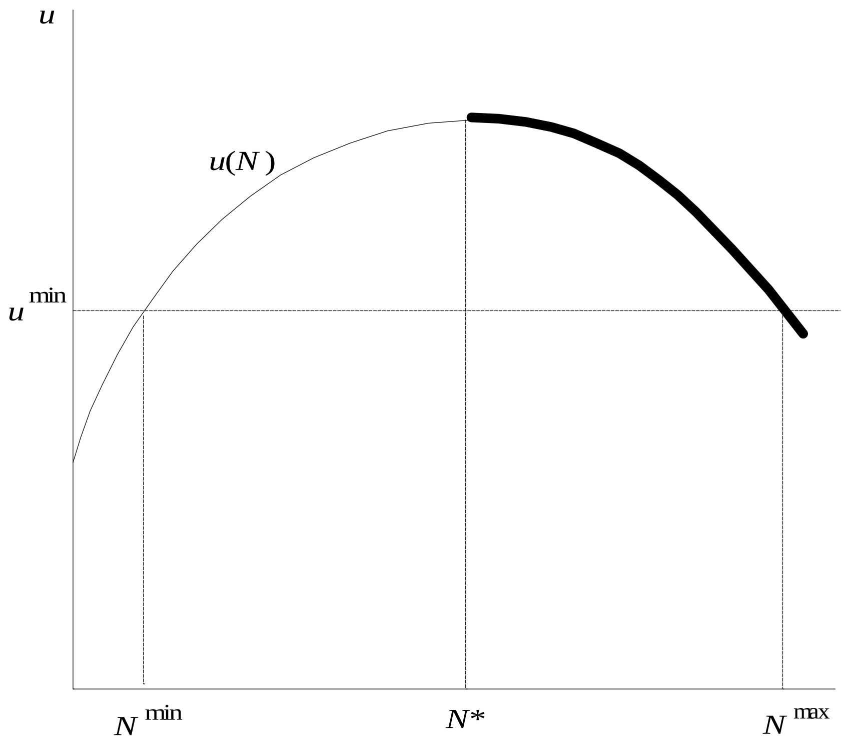
Year	Land Value (billion yen)	$\frac{\text{Land Value}}{\text{GNP}}$
1970	181,531	2.48
1975	376,406	2.54
1980	705,793	2.88
1985	1,004,073	3.09
1990	2,338,239	5.35
1993	1,855,143	4.01

Source: *National Income Accounts* (Japan Economic Planning Agency).

As shown in Chapter 2 of Kanemoto (1980), the presence of agglomeration economies tends to make the equilibrium city size too large. This can be explained by taking a simple case where the entire population is divided into homogeneous cities. Consider a country with homogeneous population where everyone lives in one of the cities. The total population is fixed at $\overline{N}$ which is divided into m cities of the same population size $N = \overline{N} / m$. In equilibrium the utility level of a household is the same regardless of where it resides. The equilibrium utility level can then be written as a function of city size, $u(N)$. An example is depicted in Figure 2, where the optimal city size is attained at N^*.

Now, it is easy to see that we have multiple equilibria in this case. The existence of agglomeration economies means that a start-up city that is initially small may not be able to achieve as high a utility level as existing large cities. For example, suppose that each new city must start at the population size of $N^{\min}$ in Figure 2. Any population size between N^* and $N^{\max}$ can then be a stable equilibrium.

Figure 2 Optimal and Market City Sizes



This can be seen as follows. Between N^* and $N^{\max}$ the utility level declines as the city size gets larger. Starting from the situation where all cities have the same population, consider a move of a resident from one city to another. The city that lost this resident now has a slightly smaller population than before and the utility of the residents rises because of the move. In contrast, the city into which the resident moved experiences a decline in the utility level. Because residents move from a city with a lower utility level to that with a higher utility level, the original equilibrium will be restored. The only way to move to another equilibrium is to start a new city. However, until the population of a city exceeds $N^{\max}$, a start-up city cannot compete with existing cities. Thus, any city size between N^* and $N^{\max}$ can remain as an equilibrium.

As noted before, cities are not homogeneous. If cities form a hierarchical structure where those at each layer of hierarchy can be regarded as homogeneous, however, a similar argument can be applied to cities in each hierarchical level to show that they tend to be too large. Because of multiplicity of equilibria, the difference between the actual and the optimum city size depends on the history. The range of possible divergence however depends on the size of the start-up city. With a hierarchical structure, a new city in a certain hierarchy usually comes from a city at one level lower. This means that it is relatively easy to increase the number of cities at a lower level of hierarchy. In contrast, it is extremely difficult to add a new city at the highest level of hierarchy. For example, the population of the Tokyo IMA is close to 32 million whereas that of the Osaka IMA is less than 15 million. It would be very difficult to move Osaka up to the level where it can compete with Tokyo at the top of hierarchy. Thus, we could conjecture that divergence from the optimal city size is larger for larger cities. Our main focus in this section is to check if this conjecture is true. In particular we compute the ratios between the total land value (minus the total social overhead capital) and the total Pigouvian subsidy for metropolitan areas at different levels of hierarchy and see whether the ratio is larger for Tokyo than other metropolitan areas.

3.3. Construction of Total Land Value Data for IMAs

The construction of the total land value data for an IMA is as follows.

First, we assume that urban land consists of urban planning areas. We therefore excludes areas outside urban planning areas in our calculation of total land values. This also means that we include urbanization control areas as well as urbanization promotion areas. Although urban development is in principle prohibited there at the moment, development in the future is expected in many of the places in the areas and it would be inappropriate to exclude them from our calculation.

Second, the land price data that we use are Chika Koji (Public Announcement of Land Prices by National Land Agency) for January 1st in 1985 and Todofuken Chika Chosa (Prefectural Land Price Survey) for July 1st in 1984 and 1985. In order to adjust for the half year difference between the two data sets, we averaged data from the latter to produce estimates for January 1st in 1985.[9]

Third, according to the urban planning law in Japan, an urban planning area is divided into urbanization promotion areas and urbanization control areas. The former is further divided into eight (8) different land use areas: type 1 exclusive residential areas, type 2 exclusive residential areas, residential areas, neighborhood commercial areas, commercial areas, semi-industrial areas, industrial areas, and exclusive industrial areas. Because industrial and exclusive industrial areas have very few samples, we combine them into one type. This gives us eight types of urban areas, i.e., urbanization control areas and seven types of land use areas in urbanization promotion areas. We compute the average land prices separately for the eight types of urban areas in each municipality. The total land value for each land use category is the average land price multiplied by the total land area in the category. The total land value for a municipality is then obtained by summing land values for all land use categories.

[9] For locations where 1984 data are not available, we used only the 1985 data.

Fourth, for some municipalities we do not have land price data to compute the average land prices. In such a case we obtain land prices by estimating the following equation for the metropolitan area.

(10) $$\ln P = a_0 + \sum_{i=1}^{7} a_i D_i + bt$$

where P, D_i's, and t are respectively land price, dummy variables for the types of urban land except for the residential areas, and distance from the city center.

3.4. The Ratio Between the Total Land Value and the total Pigouvian Subsidy

We compare the total land value and the total Pigouvian subsidy in 17 IMAs that have more than one million residents. Table 5 presents the comparison in the Cobb-Douglas case, i.e., equation (6) with the restriction of $a_3 = 0$.[10] The total land value is very high compared with the total Pigouvian subsidy in all cities. On the average the ratio between them is 145.4. According to the Henry George Theorem, this must equal the ratio between the total land value and the total differential urban rent. If we further ignore development costs and the rural rent, this ratio must equal the inverse of the user cost of capital. The user cost of capital would then equal 0.73%, which is extremely low. This may indicate that the city sizes are too large for most of the 17 cities. Our estimates of land values are quite crude, however, and substantially higher than the estimates by the Economic Planning Agency as discussed in the next section. Furthermore, the rent/value ratio is known to be very small in Japan and the user cost of 0.73% may not be overly unrealistic.

Although the total land value in the Tokyo IMA is extremely large, the total Pigouvian subsidy is also very large, and the ratio between them is slightly below the average for the 17 cities. Our evidence therefore does not support the hypothesis that Tokyo is too large. If

Tokyo were too large, then many of the other cities would also be too large. This comparison is made in terms of ratios, however, and comparison in terms of absolute magnitudes may give a completely different picture because Tokyo is more than twice as large as the second largest metropolitan area. For example, if the price/rent ratio is 120, the total land rent minus the total Pigouvian subsidy is about 1,500 billion yen for Tokyo but about 350 billion yen for Osaka. In this case both Tokyo and Osaka are too large at the current hierarchy but the difference from the optimum is much larger for Tokyo. It may then be desirable to move Osaka to the same level of hierarchy as Tokyo.

The land value/Pigouvian subsidy ratio is very high in Kyoto, Hiroshima, and Hamamatsu. This may be due to topographical reasons. For example, Kyoto is in a basin and the expansion of the city is limited by mountains.

Table 6 shows the results for the modified Cobb-Douglas case of equation (7). Because the agglomeration economies are estimated to be much smaller in this case, the ratio between the total land value and the total Pigouvian subsidy is substantially higher. If the social overhead capital is a private good, then we can use the ratio as an indicator for divergence from the Henry George Theorem. If we assume that the social overhead capital is a pure local public good, we have to subtract the value of social overhead capital from the total land value, but the ratio changes very little as indicated in the last column in Table 6. As in Table 5, the ratio for Tokyo is lower than the average and there is no indication that Tokyo is too large.

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[10] The total Pigouvian subsidy is computed from the production function estimated for 17 largest cities.

Table 5 Total Land Values and Pigouvian Subsidies: the Cobb-Douglas Case

IMA	Land Value (billion yen) (a)	Pigouvian Subsidy (billion yen) (b)	$\frac{(a)}{(b)}$	Population (1985)
Tokyo	1,031,422	7,134	144.6	31,883,659
Osaka	402,241	3,005	133.9	14,463,666
Nagoya	241,461	1,791	134.9	7,406,962
Kyoto	121,256	569	212.9	3,203,076
Sapporo	33,703	336	100.4	2,110,113
Hiroshima	59,898	355	168.6	1,988,186
Fukuoka	34,730	351	99.0	1,928,487
Kitakyushu	46,798	335	139.8	1,848,793
Sendai	25,804	170	152.2	1,579,968
Maebashi	45,055	259	174.1	1,545,802
Yokkaichi	29,884	267	111.9	1,472,053
Okayama	40,196	302	133.0	1,462,123
Kurume	21,651	220	98.3	1,243,558
Shizuoka	33,721	207	162.7	1,207,611
Utsunomiya	36,961	223	165.8	1,177,367
Hamamatsu	46,522	204	228.1	1,087,420
Kumamoto	17,189	153	112.1	1,022,891
Average			145.4	

Table 6 Total Land Values and Pigouvian Subsidies: the Modified Cobb-Douglas Case

IMA	Land Value (billion yen) (a)	Pigouvian Subsidy (billion yen) (b)	$\frac{(a)}{(b)}$	Land Value – SOC (billion yen) (c)	$\frac{(c)}{(b)}$
Tokyo	1,031,422	4,174	247.1	961,531	230.3
Osaka	402,241	1,665	241.6	372,684	223.8
Nagoya	241,461	951	253.9	225,992	237.7
Kyoto	21,256	285	426.1	115,347	405.3
Sapporo	33,703	173	194.3	26,536	153.0
Hiroshima	59,898	178	336.7	54,762	307.9
Fukuoka	34,730	172	202.0	30,711	178.6
Kitakyushu	46,798	164	285.5	43,014	262.4
Sendai	25,804	82	314.0	22,485	273.6
Maebashi	45,055	125	360.1	41,769	333.8
Yokkaichi	29,884	129	231.9	26,770	207.7
Okayama	40,196	149	269.9	36,293	243.7
Kurume	21,651	107	203.1	18,700	175.4
Shizuoka	33,721	98	343.0	31,223	317.6
Utsunomiya	36,961	106	348.5	34,403	324.4
Hamamatsu	46,522	96	482.9	44,204	458.8
Kumamoto	17,189	73	237.1	14,999	206.9
Average			292.8		267.1

4. Conclusion

The magnitudes of agglomeration economies are estimated from aggregate production functions for metropolitan areas in Japan, and the estimates are used to test the hypothesis that Tokyo is too large. In the estimation of aggregate production functions our major findings are as follows.

First, agglomeration economies are small for small cities but fairly large for cities with population larger than 200,000. Cities with population between 200,000 and 400,000 have especially large agglomeration economies: doubling the size of a city increases productivity by about 25% in the Cobb-Douglas production function case. The productivity increase is about 7% for cities with more than 400,000 residents and about 1% for cities with less than 200,000 residents.

Second, in our cross-section estimation of a metropolitan production function, the coefficient for social overhead capital is either statistically insignificant or negative. We therefore omit the variable in the estimation of the Cobb-Douglas production function. We have also estimated a modified Cobb-Douglas form in which the agglomeration effect interacts with social overhead capital in a highly nonlinear manner. Agglomeration effects are substantially smaller in this case than in the simple Cobb-Douglas case.

Using the estimates for agglomeration economies, we have tested if the Henry George Theorem for optimal city size is satisfied. We found that the total land values are very high compared with the total Pigouvian subsidies in all cities, but the ratio for Tokyo is slightly below the average for 17 largest cities in Japan. Thus, there is no evidence supporting the hypothesis that Tokyo is too large. Note however that this comparison is made in terms of ratios. Because Tokyo is much larger than other cities, the absolute difference between the differential urban rent and the Pigouvian subsidy could be extremely large.

This article is a first attempt at an empirical test of the Henry George Theorem and there is ample room for improvements. Elaboration and extension in the following directions would be useful.

First, our land value estimates are quite crude. The Economic Planning Agency publishes the total land value data, but because this data set is only at the prefectural level, we cannot use it for our study on metropolitan areas. Our land value estimates are directly comparable with the EPA's for a few prefectures that are entirely included in one of the

IMAs. In these prefectures our estimates are about three times as large as the EPA’s. One of the reasons for this difference is that we include publicly owned land (e.g., roads and parks) in our calculation because there is no reason to exclude them in the Henry George Theorem. Publicly owned land is less than a half of the total urban land, however, and this cannot explain all the differences.

Second, in our estimation of the Cobb-Douglas production function social overhead capital tends to have a negative coefficient. As noted before, the reason for this may be a simultaneous equation bias caused by the fact that relatively poor regions get larger shares of public infrastructure investment. A simultaneous equation estimation may improve our estimates.

Third, in our theoretical framework we simply assume that agglomeration economies are technological externalities. As shown by Kanemoto (1990) and Krugman (1991), a heterogeneous good model can produce urban agglomeration if it is combined with transportation and communication costs. Kanemoto (1990) showed that in such a model locational externalities emerge and Pigouvian subsidies are necessary to achieve the first best outcome. In a heterogeneous good model, however, producers are not expected to act as price takers and second best issues that are caused by price distortions complicate the analysis. The extension of the Henry George Theorem to this second best situation is left for the future.

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Figure Legends

Figure 1. The total differential urban rent and the Pigouvian subsidy

Figure 2. Optimal and Market City Sizes